# Myopia and Anchoring in Dynamic Networks: Determinacy and Persistence

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**Abstract**

Angeletos and Huo (2021) derive equilibrium and an $n$-mode representation for dynamic networks. Under their printed spectral condition, a signed three-node economy has a one-dimensional family of stationary equilibria and four persistence modes. Absolute small gain restores the $2n/n$ root count. A nonnegative feed-forward economy satisfies this condition and has a unique equilibrium, while its transfers contain poles of orders three, two, and one. Generalized modes give the replacement representation. Under nearby distinct roots, simple-mode coefficients diverge at rate $\varepsilon^{-2}$ while canceling to the repeated-pole limit. The observable-representation argument also requires a state-space realization and a separate determinacy premise.

*JEL Codes:* C32, C62, D83, D85, E32

*Keywords:* dispersed information, dynamic networks, equilibrium determinacy, repeated roots, generalized modes

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## 1. Introduction

Angeletos and Huo (2021) study how incomplete information generates myopia and anchoring in dynamic general-equilibrium models. Their multivariate extension allows the action of one group to depend on the expected actions of other groups. Proposition 10 gives a unique equilibrium and represents every group outcome as a linear combination of $n$ AR(2) components. The paper uses this representation to describe how network interactions shape persistence.

The proposition combines three mathematical steps. The interaction operator must select a stationary equilibrium. The determinant in equation (30) must have $2n$ roots inside and $n$ roots outside the unit disk. The reduced transfer functions must admit the displayed sum of $n$ simple fractions. These steps require different conditions.

The first example in this Comment concerns signed interactions. It satisfies the parameter restrictions printed before Proposition 10, including

$$\rho_{\mathrm{sp}}\Big((I-\beta)^{-1}\gamma\Big) < 1. \tag{1}$$

Its degree-nine determinant has five roots inside and four outside the unit disk. Within the stationary linear class constructed in the appendix of Angeletos and Huo (2021), the six boundary constants face five independent pole-cancellation restrictions. One free direction remains, and generic points on this line retain four observable persistence modes.

A stability repair replaces the signed comparison matrix in (1) with its entrywise absolute counterpart,

$$\rho_{\mathrm{sp}}\Big((I-\beta)^{-1}|\gamma|\Big) < 1. \tag{2}$$

Condition (2) rules out cancellation between positive and negative network paths. It gives the published $2n/n$ root split through a unit-circle homotopy and yields a weighted-norm contraction once the conditional-expectation operator is placed on the stationary process space used by the model.

The modal issue also reaches nearby systems with distinct roots. As three roots approach the repeated root in the feed-forward example, the coefficients of the simple fractions diverge at rate $\varepsilon^{-2}$ and cancel to a finite generalized-mode limit. Section 4 gives the exact identity. Thus numerical work based on separate simple modes can become ill-conditioned before the repeated-root case is reached.

The second example tests the remaining modal claim under (2). Its interaction matrix is an entrywise-nonnegative three-node feed-forward chain with spectral radius zero. The determinant has six inside roots and three outside roots, and the multiplicity-aware boundary system has a unique solution. The reduced transfers have outside pole orders three, two, and one. Three copies of a simple fraction at the same root still span one simple pole, so they cannot reproduce the cubic and quadratic poles.

The replacement result follows from the multiplicities of the reduced transfer. If the surviving denominator is $\prod_j (1 - \vartheta_j L)^{m_j}$, the group transfer takes the form

$$h_g(L) = \sum_j \sum_{r=1}^{m_j} \frac{c_{gjr}}{(1 - \vartheta_j L)^r}. \tag{3}$$

An order-$r$ term has impulse response $\binom{t+r-1}{r-1}\vartheta_j^t$. The original simple-mode formula is recovered when every pole of the reduced observable transfer has order one. Near a repeated-root configuration, a simple-mode expansion exists with coefficients that can become arbitrarily large and offset one another.

Huo and Takayama (2025) provide a later solution framework for rational-expectations models with higher-order beliefs. Their zero-count conditions and finite ARMA representation clarify the role of equilibrium selection in a broader class. The present Comment addresses a narrower source claim: it gives exact counterexamples to the unconditional network statement in Angeletos and Huo (2021), locates the missing root and multiplicity conditions in its proof, and supplies a replacement for that statement.

Table 1 separates the affected claims from the results that continue to hold. The scalar analysis, inflation application, and calibrated HANK illustration in Angeletos and Huo (2021) lie outside the refuted set.

## 2. The determinant representation

Let $\beta = \mathrm{diag}(\beta_1, \ldots, \beta_n)$ with $0 < \beta_g < 1$, let $0 < \rho < 1$, and let $\sigma_g^2 > 0$. Write

$$d_g = \frac{1}{\rho\sigma_g^2}, \qquad q_g(z) = z^2 - (\rho + \rho^{-1} + d_g)z + 1. \tag{4}$$

It is useful to keep the normalization separate from the roots. Define the normalized pencil determinant

$$\widetilde{C}(z) = \det\{B(z) - E(z)\}, \qquad B(z) = \mathrm{diag}((\beta_g - z)q_g(z)), \qquad E(z) = z\,\mathrm{diag}(d_g)\gamma. \tag{5}$$

The determinant $C_A(z)$ of the appendix matrix $A(z)$ satisfies $C_A(z) = \kappa\widetilde{C}(z)$ for a nonzero scalar $\kappa$ under this normalization. Both determinants have the same roots and multiplicities. They have degree $3n$. The appendix seeks a causal stationary transfer by choosing $2n$ boundary constants to cancel the inside roots. Its formula then associates the inverses of the $n$ outside roots with endogenous persistence parameters.

TABLE 1. Proposition 10: claims, examples, and additional conditions

| Component | Example | Consequence | Sufficient repair |
|---|---|---|---|
| Stationary determinacy | Signed rational network | One-dimensional stationary family | Absolute small gain and an explicit process-space contraction |
| Root count | Signed rational network | Five inside and four outside roots for $n = 3$ | Unit-circle exclusion under absolute small gain |
| Simple modal form | Nonnegative feed-forward network | Pole orders 3, 2, 1 | Simple reduced poles or generalized modes |
| Inside cancellation | Repeated inside root | Value equations omit multiplicity restrictions | Hermite derivative conditions |
| Proposition 11 proof | Both networks | Diagonal modal construction lacks its stated basis | Generalized state-space realization and a separate determinacy condition |

This construction places two demands on the determinant. Its inside/outside split must match the number of boundary constants. Its surviving factors must support the claimed partial-fraction form. Algebraic multiplicity enters both demands.

## 3. Signed interactions and equilibrium determinacy

Consider three groups and set

$$\beta = \frac{2}{3}I, \qquad \rho = \frac{2}{3}, \qquad (\sigma_1^2, \sigma_2^2, \sigma_3^2) = \left(\frac{3}{8}, 15, \frac{3}{8}\right), \tag{6}$$

$$\gamma = \begin{pmatrix} 0 & -4/5 & 1 \\ 1/5 & 0 & 0 \\ 1/5 & 0 & 0 \end{pmatrix}. \tag{7}$$

The associated private-signal filter roots are $(1/6, 3/5, 1/6)$. Each row of $\delta = \beta + \gamma$ sums to $13/15$, so the action weights lie in the interior of the range imposed in the network section. Since $(I - \beta)^{-1} = 3I$, the characteristic polynomials of the signed and absolute comparison matrices are

$$\det(xI - 3\gamma) = x(x^2 - 9/25), \qquad \det(xI - 3|\gamma|) = x(x^2 - 81/25). \tag{8}$$

The two spectral radii are 3/5 and 9/5. The example satisfies (1) and violates (2).

PROPOSITION 1 (Signed network). *For the economy in* (6)–(7), $\widetilde{C}$ *and* $C_A$ *have five roots inside and four roots outside the unit disk. All nine roots are simple. The pole-cancellation system in the published appendix has rank five in its six boundary constants. Within that stationary linear solution class, the equilibrium set contains an affine line, and generic points on the line retain four outside poles.*

PROOF. Exact rational expansion of (5), followed by primitive integer scaling, gives

$$\begin{aligned} C_0(z) = &-108000 + 2062800z - 14512752z^2 + 54186872z^3 - 115881318z^4 \\ &+ 143227953z^5 - 99554400z^6 + 36273825z^7 - 6050700z^8 + 364500z^9. \end{aligned} \tag{9}$$

The greatest common divisor of $C_0$ and $C_0'$ equals one. Apply the Cayley map $w = (z - 1)/(z + 1)$, which sends the unit disk to the left half-plane. The transformed primitive polynomial has ascending coefficients

$$\begin{aligned} &(2195, -180915, -1455489, 1501273, 27029589, 23579979, \\ &-84779891, -112716117, 75619596, 118055780). \end{aligned} \tag{10}$$

Its exact rational Routh array has first-column signs $(-, -, -, +, -, -, -, -, +, -)$ and no zero pivot or zero row. Four sign changes place four roots in the right half-plane, hence four roots of $C_0$ outside the unit disk. The other five roots are inside. Substitution in the Cramer numerators derived from equation (A18) gives five independent cancellation equations in six constants. A nonzero $5 \times 5$ minor establishes rank five. Along the resulting null direction, each outside-root numerator is a nonzero affine function of the free coordinate. Excluding their finitely many zeros leaves four surviving poles. A fourth-order Hankel determinant is nonzero at two points on this line, so the corresponding transfer cannot be generated by three exponential modes. Appendix A gives the supporting algebra. □

The count in Proposition 1 has the direct dimension implication used in the appendix construction. Six constants remove five stable poles, leaving one boundary degree of freedom. Four unstable roots remain in the denominator of generic reduced transfers, giving four stable persistence parameters after inversion. The example shows that the printed spectral restriction does not provide the uniqueness premise used by the appendix to infer the root count.

## 4. A nonnegative network with generalized modes

The signed example points toward absolute small gain. The next example satisfies that condition and isolates the modal step. Set

(11)

$$\beta = \frac{1}{3}I, \qquad \rho = \frac{2}{3}, \qquad \sigma_g^2 = \frac{9}{7} \quad (g = 1, 2, 3), \qquad \phi = (1, 1, 1)', \qquad \gamma = \begin{pmatrix} 0 & 1/6 & 0 \\ 0 & 0 & 1/6 \\ 0 & 0 & 0 \end{pmatrix}.$$

The matrix $\gamma$ is entrywise nonnegative and satisfies $\gamma^3 = 0$. Both comparison radii in (1) and (2) equal zero. Each row sum of $\delta$ is at most 1/2.

PROPOSITION 2 (Repeated persistence root). *For the economy in* (11)*,*

$$\widetilde{C}(z) = -\frac{(z-3)^3(3z-1)^6}{729}, \qquad C_A(z) = -\frac{(z-3)^3(3z-1)^6}{19683} = \frac{\widetilde{C}(z)}{27}. \tag{12}$$

*The Hermite cancellation system has a unique solution. The reduced group transfers are*

$$h_1(z) = -\frac{27(14115382z^2 - 85874907z + 130672719)}{179830784(z-3)^3}, \tag{13}$$

$$h_2(z) = -\frac{81(650z - 1999)}{25088(z-3)^2}, \tag{14}$$

$$h_3(z) = -\frac{27}{14(z-3)}. \tag{15}$$

*The first and second transfers are outside the simple-mode representation displayed in Proposition 10.*

PROOF. The matrix $B(z) - E(z)$ is upper triangular, and its diagonal entries share the scalar factor $-(z-3)(3z-1)^2/9$. Their product gives the first equality in (12); the appendix normalization supplies the factor 1/27 in the second. Cancellation of the inside root 1/3 with multiplicity six requires the first six derivatives of each Cramer numerator to vanish there. The resulting eighteen equations in six constants have rank six and the unique solution recorded in Appendix B. Direct substitution gives (13)–(15). Their numerators are nonzero at $z = 3$, so their pole orders are three, two, and one. Proposition 10 associates each outside root with a simple fraction. All three outside roots here equal 3, so every such fraction is proportional to $(1 - z/3)^{-1}$. A finite sum of those fractions has pole order one. Equations (13) and (14) have higher orders. □

The pole pattern follows from the network topology. For a homogeneous scalar diagonal factor $s$ and scalar $k$,

$$(sI - k\gamma)^{-1} = \frac{I}{s} + \frac{k\gamma}{s^2} + \frac{k^2\gamma^2}{s^3}. \tag{16}$$

Every three-node chain with two nonzero links has the same three denominator orders. The exact boundary conditions in Proposition 2 leave the highest-order terms visible.

A nearby diagonal perturbation splits the common factors into $s - k\varepsilon$, $s - 2k\varepsilon$, and $s - 3k\varepsilon$. The length-two path term has the identity

$$\frac{abk^2}{(s-k\varepsilon)(s-2k\varepsilon)(s-3k\varepsilon)} = \frac{ab}{2\varepsilon^2}\frac{1}{s-k\varepsilon} - \frac{ab}{\varepsilon^2}\frac{1}{s-2k\varepsilon} + \frac{ab}{2\varepsilon^2}\frac{1}{s-3k\varepsilon}. \tag{17}$$

The simple-mode coefficients diverge at rate $\varepsilon^{-2}$ and cancel to the finite repeated-pole limit. This calculation links the exact repeated-root example to nearby systems with distinct roots.

## 5. Corrected stability and representation results

This section gives separate conditions for unit-circle exclusion and the root count, equilibrium selection, inside-root cancellation, and the final transfer representation.

THEOREM 1 (Unit-circle exclusion and root count). *Suppose* $0 < \beta_g < 1$, $0 < \rho < 1$, $\sigma_g^2 > 0$, *and* (2) *holds. The polynomial* $\widetilde{C}$ *in* (5)*, and hence* $C_A$*, has no root on the unit circle. It has exactly* $2n$ *roots inside and* $n$ *roots outside the unit circle, counted with algebraic multiplicity.*

PROOF. For $|z| = 1$, equation (4) gives

$$|q_g(z)| = |z|\,|z + z^{-1} - (\rho + \rho^{-1} + d_g)| \geq d_g, \qquad |\beta_g - z| \geq 1 - \beta_g. \tag{18}$$

Hence the entries of $B(z)^{-1}E(z)$ satisfy

$$|B(z)^{-1}E(z)| \leq (I - \beta)^{-1}|\gamma|. \tag{19}$$

The comparison theorem for nonnegative matrices and (2) imply $\rho_{\mathrm{sp}}(B(z)^{-1}E(z)) < 1$. Thus $I - B(z)^{-1}E(z)$ is invertible and $\widetilde{C}(z) \neq 0$ on the unit circle. The same bound applies to the homotopy $E_t(z) = tE(z)$ for $t \in [0,1]$, so the root count is constant along the homotopy. At $t = 0$, each factor $(\beta_g - z)q_g(z)$ has the root $\beta_g$ and the smaller reciprocal root of $q_g$ inside the unit disk, together with the larger reciprocal root outside. The count is $2n/n$. □

PROPOSITION 3 (Process-space contraction). *Let* $\mathcal{H}_g$ *be the stationary square-integrable process space for group* $g$*, and let* $\mathcal{H} = \prod_g \mathcal{H}_g$*. Suppose the aggregate best-response map* $F : \mathcal{H} \to$

*$\mathcal{H}$ obeys*

$$\|F_g(a) - F_g(b)\|_g \leq \sum_k S_{gk}\|a_k - b_k\|_k, \qquad S = (I - \beta)^{-1}|\gamma|. \tag{20}$$

*Under* (2)*, $F$ has a unique fixed point in $\mathcal{H}$.*

PROOF. Set $w = (I - S)^{-1}\mathbf{1}$. Then $w > 0$ and $Sw = w - \mathbf{1} < w$. Equip $\mathcal{H}$ with $\|a\|_w = \max_g \|a_g\|_g / w_g$. Equation (20) yields $\|F(a) - F(b)\|_w \leq \kappa\|a - b\|_w$ for $\kappa = \max_g (Sw)_g / w_g < 1$. The contraction theorem gives a unique fixed point. □

To apply Proposition 3, let $\mathcal{E}_{g,t}X = \int_0^1 E_{i,g,t}[X]\, di$ denote the group-average forecast operator. Forward substitution of equation (27) in Angeletos and Huo (2021) gives the aggregate best-response map

$$[F_g(a)]_t = \mathcal{E}_{g,t} \sum_{s=0}^{\infty} \beta_g^s \left( \phi_g \xi_{t+s} + \sum_k \gamma_{gk} a_{k,t+s+1} \right). \tag{21}$$

Conditional expectation is an $L^2$ contraction, integration over agents has operator norm at most one by Minkowski's inequality, and time shifts preserve the norm of a stationary process. Applying these bounds to the difference between two inputs in (21) yields $\|F_g(a) - F_g(b)\|_g \leq (1 - \beta_g)^{-1} \sum_k |\gamma_{gk}| \|a_k - b_k\|_k$, which is (20). This derivation states the process-space premise used to apply the contraction result to equation (27).

LEMMA 1 (Multiplicity-aware cancellation). *Let $N$ be a polynomial over a field of characteristic zero. The factor $(z - \zeta)^m$ divides $N$ exactly when*

$$N^{(r)}(\zeta) = 0, \qquad r = 0, \ldots, m - 1. \tag{22}$$

PROOF. Expand $N$ in its finite Taylor series around $\zeta$. The first $m$ coefficients vanish exactly when the expansion begins at order $m$. □

The appendix proof of Proposition 10 writes value equations at the listed inside roots. Repetition of the same value equation leaves the derivative restrictions in (22) unrecorded. The eighteen-equation calculation used in Proposition 2 supplies those restrictions and produces the unique boundary vector.

THEOREM 2 (Generalized persistence modes). *Suppose inside-root cancellation produces a strictly proper real rational transfer*

$$h_g(L) = \frac{P_g(L)}{D_g(L)}, \qquad D_g(L) = c_g \prod_{j=1}^{J_g} (1 - \vartheta_{gj} L)^{m_{gj}}, \qquad |\vartheta_{gj}| < 1, \tag{23}$$

*where the $\vartheta_{gj}$ are distinct and may be complex. Unique complex coefficients $c_{gjr}$ exist such that*

$$h_g(L) = \sum_{j=1}^{J_g} \sum_{r=1}^{m_{gj}} \frac{c_{gjr}}{(1-\vartheta_{gj}L)^r}. \tag{24}$$

*For a real transfer, terms at conjugate roots pair to give real rational terms and real impulse responses. The impulse response of the order-r term is $\binom{t+r-1}{r-1}\vartheta_{gj}^t$.*

PROOF. The primary factors in (23) are pairwise coprime. The partial-fraction theorem gives (24) and uniqueness. The negative-binomial power series gives the impulse-response coefficient. □

COROLLARY 1 (Simple reduced poles). *If every pole of each reduced observable transfer has order one, equation* (24) *reduces to a sum of simple persistence components. Repeated determinant roots are compatible with this corollary when numerator cancellation leaves every reduced pole simple.*

The corollary states the needed property at the level of observable transfers. In a state-space formulation, a semisimple minimal realization with a complete observable eigenbasis gives an equivalent route. The determinant root count gives algebraic multiplicity, while cancellation in the transfer determines the reduced pole orders.

## 6. Implication for Proposition 11

Proposition 11 of Angeletos and Huo (2021) represents the incomplete-information equilibrium through a complete-information system. Its appendix proof begins with the simple $n$-mode expression from Proposition 10, stacks the mode loadings in a square matrix $Q$, and uses $Q\Lambda Q^{-1}$. Both examples interrupt that route. The signed example has four surviving persistence roots for three groups and a stationary equilibrium family. The positive example requires a generalized-mode state with a Jordan transition block.

Together, these results locate the two additional requirements in the published proof. A complete-information rationalization for a selected equilibrium requires a state-space argument. Equilibrium determinacy requires an independent premise under the printed signed condition, while the positive repeated-root example requires a generalized-state construction in place of the diagonal $Q\Lambda Q^{-1}$ step.

A replacement statement can start from a minimal observable realization

$$x_t = Jx_{t-1} + D\xi_t, \qquad a_t = Hx_t, \tag{25}$$

where $J$ contains the generalized persistence blocks in Theorem 2. A first-order law in the observable vector $a_t$ follows when the observable loading map supplies the required invertible realization. Selection of a unique equilibrium remains governed by Proposition 3 or another stated determinacy condition.

## 7. Conclusion

The network result in Angeletos and Huo (2021) uses one spectral restriction to support equilibrium selection, a root count, and a simple modal description. The signed witness separates the first two properties from the published restriction. The nonnegative feed-forward witness separates the modal property from the absolute-small-gain repair. Theorems 1 and 2 provide a usable division: absolute small gain controls the determinant on the unit circle, Hermite conditions remove inside roots with their multiplicities, and generalized partial fractions describe the surviving dynamics. Proposition 11 then requires an observable state realization and a separately stated determinacy condition.

## References


Angeletos, George-Marios and Zhen Huo. 2021. "Myopia and Anchoring." *American Economic Review* 111 (4): 1166–1200. `10.1257/aer.20191436`.

Huo, Zhen and Naoki Takayama. 2025. "Rational Expectations Models with Higher-Order Beliefs." *Review of Economic Studies* 92 (5): 3138–3173. `10.1093/restud/rdae096`.


## Appendix A. Algebra for the signed witness

The rational-filter identity for the three signal roots is

$$\lambda_g + \lambda_g^{-1} = \rho + \rho^{-1} + (\rho\sigma_g^2)^{-1}, \tag{A1}$$

and direct substitution gives zero residual in all three groups. The comparison radii follow from (8). Exact rational expansion gives the determinant coefficients in (9).

The Cayley-transformed coefficients in (10) generate an exact rational Routh table. Its first column has no zero element, and the array has no zero row. The four sign changes therefore give the complete outside-root count. Rational isolating intervals place all nine roots strictly away from the unit circle.

Let $b \in \mathbb{R}^6$ denote the boundary constants in equation (A18), and let $M_{\text{in}} b = v_{\text{in}}$ collect the five inside-root cancellation equations. Exact symbolic reduction gives $\operatorname{rank}(M_{\text{in}}) = 5$. A $5 \times 5$ minor is nonzero. Write the solution line as $b = b_0 + \tau v$. At every outside root $z_j$, each Cramer numerator is affine in $\tau$. At $\tau = 0$ and $\tau = 1$, interval evaluation places every outside-root numerator and each fourth-order Hankel determinant away from zero, excluding a representation with three exponential modes.

## Appendix B. Algebra for the nonnegative witness

For (11), each group has $d_g = 7/6$ and smaller signal-filter root $1/3$. The normalized matrix in (5) is triangular, with diagonal entry $-(z-3)(3z-1)^2/9$ and upper-chain entry $-7z/36$. The appendix matrix uses a groupwise normalization by $1/3$, giving diagonal entry $-(z-3)(3z-1)^2/27$, upper-chain entry $-7z/108$, and determinant factor $1/27$. These calculations give both equalities in (12).

Let the six boundary constants be $(x_1, x_2, x_3, y_1, y_2, y_3)$. Applying Lemma 1 to the three Cramer numerators yields eighteen equations. Their exact rank and augmented rank are six, and the solution is

(A2) $$(x_1, x_2, x_3, y_1, y_2, y_3) = \left(\frac{130672719}{179830784}, \frac{17991}{25088}, \frac{9}{14}, \frac{433107}{6422528}, \frac{27}{448}, 0\right).$$

All eighteen derivative residuals vanish exactly. Substitution into equation (A18) gives zero residual in each group and reduces the transfers to (13)–(15).

In the basis $u = 1 - z/3$, the generalized decompositions are

(A3) $$h_1(z) = \frac{63519219}{89915392}\frac{1}{u} + \frac{72405}{3670016}\frac{1}{u^2} + \frac{63}{131072}\frac{1}{u^3},$$

(A4) $$h_2(z) = \frac{8775}{12544}\frac{1}{u} + \frac{9}{512}\frac{1}{u^2},$$

(A5) $$h_3(z) = \frac{9}{14}\frac{1}{u}.$$

Each identity has exact zero residual after multiplication by the common denominator.