A Model of Online Misinformation: Homophily and Reachability

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Abstract

Acemoglu, Ozdaglar, and Siderius (2024) show that greater homophily lowers engagement for sufficiently reliable content when each agent's expected degree is held fixed. In their high-reliability branch, every reached agent shares, so engagement equals expected network reach. A four-agent island network reverses the stated comparison. Under the expected-degree expression used in the proof, raising the within-island link probability from 9/10 to 99/100 and lowering the cross-island probability from 7/10 to 61/100 raises engagement from 0.983116 to 0.9864366. The reversal holds along a continuous fixed-degree family. The proof compares links between islands while the degree-preserving change also alters reach within each island. A replacement condition compares the pivotal reach of the two edge classes. At the reliability-one endpoint, full connectivity attains maximal engagement.

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1. Introduction

Acemoglu, Ozdaglar, and Siderius (2024) study content sharing on a stochastic network chosen by a social-media platform. Their Theorem 2 separates two effects of homophily. Greater within-community exposure can encourage the sharing of unreliable content, while lower cross-community exposure can reduce circulation. Holding every user's expected degree fixed, part (b) states that the circulation effect determines engagement when reliability is sufficiently high.

At high reliability, the most-sharing equilibrium has every reached agent share. The problem then becomes a question about the expected size of the seed's reachable set. Expected degree fixes the mean number of outgoing links. Multi-step reach also depends on where those links point and whether they connect a reached vertex to a new vertex. A degree-preserving increase in homophily raises within-island reach while it lowers transmission between islands.

This Comment gives an exact four-agent comparison. Both link probabilities remain interior, the islands have equal size, and the product law is the one used in the published proof. Engagement rises under greater homophily along the expected-degree path printed in that proof. The same conclusion holds under the non-self degree convention generated by realized outgoing neighborhoods. For the four-agent model, a polynomial inequality gives the exact local sign. For a finite island network, the corresponding condition compares the pivotal-reach contributions of within- and cross-island edges.

The correction concerns Theorem 2(b) and the circulation interpretation attached to it. The low-reliability result in Theorem 2(a) uses the effect of homophily on sharing incentives and lies outside the example. The example also leaves Theorem 3(ii) outside its refuted set; at $r = 1$, a fully connected network reaches the entire population and maximizes engagement. Gong and Yang (2025) obtain increasing reach from homophily in a different asymptotic contagion model. The calculation below locates that possibility inside the finite all-share branch of Acemoglu, Ozdaglar, and Siderius (2024).

2. The high-reliability comparison

There are N agents. A matrix $P = (p_{ij})$ gives the probabilities of directed outgoing links, and a sharing agent passes the article to every realized outgoing neighbor. In an island network, $p_{ij} = p_s$ for agents on the same island and $p_{ij} = p_d$ for agents on different islands, with $p_s \geq p_d$. Greater homophily raises p_s, lowers p_d, and preserves every agent's expected degree. The proof of Theorem 2 uses

$$1 - (1 - p_d)^{N_\ell N_{\ell'}} \tag{1}$$

for the probability of at least one directed link from island ℓ to island ℓ'. Equation (1) fixes the independent Bernoulli link law used here.

The paper assumes that an article with reliability $r = 1$ is truthful with probability one. Disliking then yields $-\tilde{c} < 0$ and is dominated by ignoring. Once dislikes are excluded, sharing yields $u + \kappa S_i > 0$.

LEMMA 1 (All-share reduction). *At $r = 1$, every reached agent shares. Platform engagement equals $N^{-1}\mathbb{E}|\mathcal{R}(i^*)|$, where $\mathcal{R}(i^*)$ is the set of vertices reachable from the seed i^* through realized outgoing links.*

PROOF. Truth is certain at $r = 1$. Dislike is strictly dominated by ignore, so no agent dislikes. With $D_i = 0$, share yields $u + \kappa S_i > 0$ and is strictly preferred to ignore. Diffusion therefore continues along every realized outgoing link from a reached vertex. The agents who share are precisely the vertices in $\mathcal{R}(i^*)$. □

Theorem 2 quantifies over every $r > \bar{r}$, where $\bar{r} < 1$. The first step of its proof makes the most-sharing equilibrium all-share for every sharing network throughout this interval. Conditional on that step, engagement equals expected reach and is independent of r. The network comparison below therefore applies for every $r > \bar{r}$. Lemma 1 gives a direct verification at $r = 1$.

3. A four-agent island network

Let the two islands be $\{0, 1\}$ and $\{2, 3\}$, and take agent 0 as the seed. The islands have equal size, so every seed has the same expected reach. The proof writes an island-ℓ agent's expected degree as $N_\ell p_s + (N - N_\ell) p_d$. In this example its degree-preserving path is

$$\delta_{\mathrm{pr}}(p_s, p_d) = 2p_s + 2p_d. \tag{2}$$

Self-links play no role in a realized outgoing neighborhood, which gives the alternative count $\delta_{\mathrm{ns}}(p_s, p_d) = p_s + 2p_d$. Both conventions are examined below.

Write $s = p_s$ and $d = p_d$. Enumerating the reachable set gives the expected number of reached agents

$$\begin{aligned} R(s, d) = &- 6d^4s^2 + 7d^4s - 3d^4 + 14d^3s^2 - 8d^3s + 2d^3 \\ &- 10d^2s^2 - 4d^2s + 2d^2 + 2ds^2 + 4ds + 2d + s + 1. \end{aligned} \tag{3}$$

Let $a(A)$ be the probability that the seed reaches every vertex of the subgraph induced by

A, for $0 \in A$. It obeys

$$a(A) = 1 - \sum_{\substack{B \subsetneq A \\ 0 \in B}} a(B) \prod_{i \in B} \prod_{j \in A \setminus B} (1 - p_{ij}). \tag{4}$$

The event that A is exactly the reachable set has probability

$$a(A) \prod_{i \in A} \prod_{j \notin A} (1 - p_{ij}). \tag{5}$$

Summing $|A|$ times (5) over the eight subsets containing the seed and collecting powers of s and d gives (3).

PROPOSITION 1 (Fixed-degree reversal). *Consider the change*

$$(p_s, p_d) = \left(\frac{9}{10}, \frac{7}{10}\right) \quad \longrightarrow \quad (p_s, p_d) = \left(\frac{99}{100}, \frac{61}{100}\right). \tag{6}$$

It is an increase in homophily that preserves the expected degree in (2). *Throughout the all-share region $r > \bar{r}$, engagement rises by* 6641249427/2000000000000.

PROOF. Both pairs satisfy $p_s \geq p_d$, the within-island probability rises, and the cross-island probability falls. Equation (2) gives

$$2\frac{9}{10} + 2\frac{7}{10} = 2\frac{99}{100} + 2\frac{61}{100} = \frac{16}{5}. \tag{7}$$

Substitution into (3) gives

$$R\left(\frac{9}{10}, \frac{7}{10}\right) = \frac{245779}{62500}, \qquad R\left(\frac{99}{100}, \frac{61}{100}\right) = \frac{1972873249427}{500000000000}. \tag{8}$$

Dividing by four gives

$$\mathcal{E}\left(\frac{9}{10}, \frac{7}{10}\right) = \frac{245779}{250000} = 0.983116, \qquad \mathcal{E}\left(\frac{99}{100}, \frac{61}{100}\right) = \frac{1972873249427}{2000000000000} = 0.9864366247.$$

Their difference is the stated positive fraction. □

Under the non-self count $p_s + 2p_d$, the comparison $(3/5, 11/20) \longrightarrow (19/20, 3/8)$ preserves expected degree at 17/10 and raises engagement from 1745417/2000000 = 0.8727085 to 118479/131072 = 0.9039230347. The reversal therefore survives the degree convention generated directly by realized outgoing neighborhoods.

4. A sign condition and replacement

Along the degree-preserving path in (2), $\mathrm{d}d/\mathrm{d}s = -1$. Differentiating (3) gives

$$\frac{\mathrm{d}R}{\mathrm{d}s} = (d-1)C(s,d), \tag{9}$$

where

$$\begin{aligned} C(s,d) = {} & 24s^2d^2 - 18s^2d + 2s^2 - 12sd^3 - 12sd^2 - 8sd + 4s \\ & + 7d^3 + 11d^2 + d + 1. \end{aligned} \tag{10}$$

For $d < 1$, $C(s,d) < 0$ defines an open region in which greater homophily raises engagement. Along the fixed-degree family $d = 8/5 - s$, substitution gives $C(s, 8/5 - s) = Q(s)/125$, where $Q(s) = 4500s^4 - 16925s^3 + 27225s^2 - 22329s + 7429$. On $[9/10, 1]$, Q decreases from $Q(9/10) = -29/40$, so the derivative is positive throughout this interval. Proposition 1 uses two interior points from the family. The published local sign holds where $C(s,d) \geq 0$.

For the non-self count, $\mathrm{d}d/\mathrm{d}s = -1/2$ and

$$\frac{\mathrm{d}R}{\mathrm{d}s} = (s-d)(d-1)B(s,d), \qquad B(s,d) = 12sd^2 - 9sd + s - 7d^2 - 5d + 2. \tag{11}$$

For $0 < d < s < 1$, $B(s,d) < 0$ gives the reversal and $B(s,d) \geq 0$ gives the local published direction.

The same calculation has a finite-network form. For a realized directed graph G and an edge e, let

$$\Delta_e(G) = |\mathcal{R}_{G\cup\{e\}}(i^*)| - |\mathcal{R}_{G\setminus\{e\}}(i^*)|. \tag{12}$$

The expectation is taken over all edges other than e. Define the aggregate within- and cross-island pivotal reach by

$$I_s = \sum_{e \text{ within}} \mathbb{E}[\Delta_e(G)], \qquad I_d = \sum_{e \text{ across}} \mathbb{E}[\Delta_e(G)]. \tag{13}$$

For equal islands of size m, an agent's expected degree is $(m-1)p_s + (N-m)p_d$. A degree-preserving local increase in p_s therefore satisfies

$$\frac{\mathrm{d}p_d}{\mathrm{d}p_s} = -\frac{m-1}{N-m}. \tag{14}$$

Since expected reach is multilinear in the edge probabilities, differentiation edge by edge yields

$$\frac{\mathrm{d}}{\mathrm{d}p_s}\mathbb{E}|\mathcal{R}(i^*)| = I_s - \frac{m-1}{N-m}I_d. \tag{15}$$

Thus the high-reliability decline follows locally under

$$I_s \le \frac{m-1}{N-m}I_d. \tag{16}$$

Condition (16) compares the number of new vertices delivered by a link, while degree preservation compares only the number of links. In the four-agent model under the non-self count, evaluating (15) reduces to (11). Under the expected-degree path printed in the proof, the corresponding expression is (9).

5. The proof and the retained results

The proof of Theorem 2(b) first invokes all-share and expected component size. It then selects a seed from the largest island, states that its agents are connected to all agents on their own island, and replaces cross-island link probabilities with the island-connection probabilities in (1). A coupling of cycles across islands completes the argument.

The island definition permits $p_s < 1$. A reached vertex can therefore leave members of its own island unreached. It can also enter another island without reaching the rest of that island. The degree-preserving comparison raises p_s, so the omitted within-island channel moves in the opposite direction from the cross-island deletion. Proposition 1 belongs to a continuous family where the first movement dominates.

Theorem 2(a) has a separate incentive channel: at low reliability, agents outside the seed island dislike the article and higher homophily changes equilibrium sharing within the seed island. The example leaves that result open. Theorem 3(ii) compares platform choices at high reliability. At $r = 1$, full connectivity, $p_s = p_d = 1$, reaches all N agents and maximizes engagement. The example therefore preserves this endpoint and leaves the rest of Theorem 3(ii) open.

6. Conclusion

In the all-share branch, homophily changes two margins of network reach. Lower cross-island probability removes routes between communities, and higher within-island probability expands access inside every visited community. Equal expected degree leaves their pivotal effects unrestricted. A four-agent network reverses Theorem 2(b), and the reversal occupies an open parameter region. The pivotal-reach condition in (16) supplies the corresponding finite-network premise for the published direction.

References

Acemoglu, Daron, Asuman Ozdaglar, and James Siderius. 2024. "A Model of Online Misinformation." *Review of Economic Studies* 91 (6): 3117–3150. `10.1093/restud/rdad111`.

Gong, Qiang and Huanxing Yang. 2025. "Homophily and Spread of Misinformation in Random Networks." *Economic Theory* 79 (4): 1311–1349. `10.1007/s00199-024-01619-z`.