

BETWEEN MODELS: COUNTERFACTUAL FRONTIERS ACROSS CAUSAL-RESPONSE EXPERIMENTS

Chae-Yeon Xon
Yonsei University

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WHAT IS THE ECONOMIC QUESTION?

When do several structural models support the same policy conclusion?

- Model families can fit a common response experiment while implying different policy signs.
- Agreement depends on the response protocol, target, weighting, and structural domain.
- The paper compares family coverage with the first policy-sign reversal.

HOW SHOULD A POSITIVE OR NEGATIVE GAP BE READ?

- If family coverage requires radius 1.0 and the first sign reversal occurs at 1.4, the gap is 0.4.
- Radii between 1.0 and 1.4 cover the family while preserving the common sign.
- If a refitted experiment moves the first reversal to 0.8, the gap becomes negative and that joint conclusion disappears.

WHAT MUST BE COMMON ACROSS MODEL FAMILIES?

- A response protocol defines the shock path, horizons, observables, and normalization.
- A response metric and policy target define distance and sign.
- An evaluated structural domain specifies which family members enter the comparison.

WHAT SINGLE QUANTITY ORGANIZES THE FRONTIER?

$$\Delta = c_{\text{sign}} - c_{\text{entry}}$$

- c_{entry} is the radius needed to cover every declared family.
- c_{sign} is the first radius at which the target sign can reverse.
- $\Delta > 0$ leaves a coverage-and-sign interval; $\Delta \leq 0$ closes it.

WHEN CAN FAMILY COVERAGE AND SIGN AGREEMENT COEXIST?

Proposition (Coverage-and-reversal frontier)

Compare the radius needed to cover the declared model family with the radius to the first policy-sign reversal. A positive gap gives a range in which family coverage and sign agreement hold together.

The gap is conditional on the response protocol, metric, domain, and target.

WHERE CAN CROSS-MODEL AGREEMENT FAIL?

Common experiment



Family response fibers



Policy sign

WHEN DOES A POLICY FEATURE FACTOR THROUGH OBSERVED RESPONSES?

Theorem (Global fiber-factorization characterization)

A policy feature is identified from the declared response map exactly when it is constant on every response fiber; equivalently, it factors through the attained response representation.

The result turns cross-model identification into a statement about observational fibers.

HOW DOES AN EXPERIMENT CHANGE THE SIGN MARGIN?

Proposition (Experiment-stable sign separation)

For each experiment, an all-family common-negative-sign interval exists exactly when its coverage-to-reversal gap is positive. Every experiment in a declared collection admits such an interval when the infimum gap is positive; one nonpositive gap rules out collection-wide sign separation.

Experiment stability is therefore assessed across the entire declared collection of response constructions.

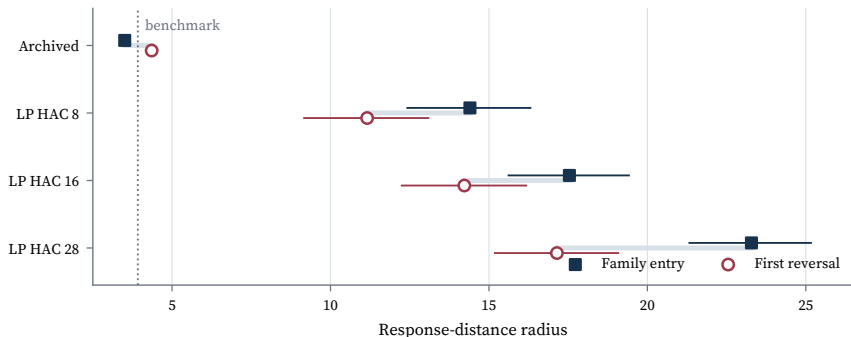
WHICH CONCLUSION SURVIVES ACROSS MODEL FAMILIES?

- Family coverage and policy-sign stability are recorded by two distinct radii.
- Their gap gives the interval in which every declared family is covered while the sign remains common.
- Repeating the calculation across empirical experiments reveals whether that conclusion belongs to the policy question or to one response protocol.

WHAT MONETARY EXPERIMENTS ARE COMPARED?

- The baseline structural comparison uses four model families under one response protocol.
- A lag-augmented local-projection refit supplies an alternative empirical response object.
- Common-support exercises vary weighting, shock-path profiling, and calendar estimation.

DOES THE SIGN-SEPARATION GAP SURVIVE ALTERNATIVE EXPERIMENTS?



Main-text experiment-stability comparison.

WHAT DO THE MONETARY COMPARISONS FIND?

- The baseline four-family structural support has a positive gap.
- The lag-augmented local-projection refit has negative gaps under three covariance specifications.
- Positive target crossings are small, and calendar re-estimation produces gaps on both sides of zero.

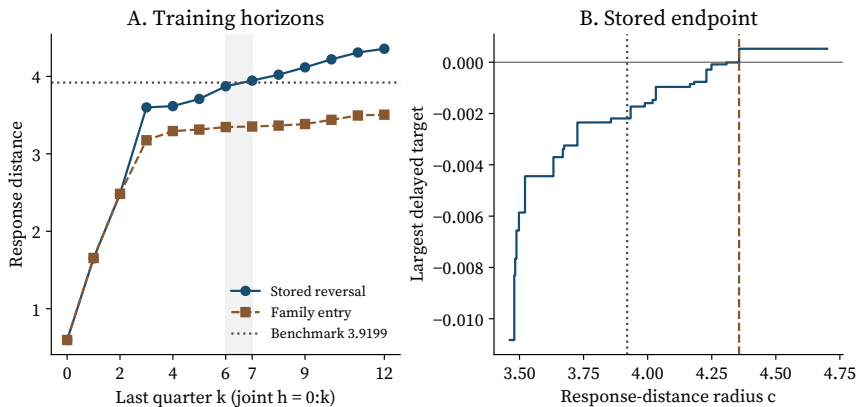
HOW IS RESPONSE-ESTIMATION UNCERTAINTY CARRIED FORWARD?

- Uniform projection inference treats model responses as generated objects.
- Covariance specifications and response weighting are part of the reported protocol.
- Evidence increments show how additional response blocks refine the fiber comparison.

WHY DO ALTERNATIVE EMPIRICAL RESPONSES CHANGE THE ANSWER?

- Shock paths and horizon weights change distances between model and empirical responses.
- Covariance estimates change the metric used for the uncertainty-adjusted comparison.
- Calendar re-estimation changes the empirical center of the response fiber.

WHERE DO COVERAGE AND REVERSAL RADII CROSS?



Appendix frontier paths across response comparisons.

HOW IS A COMMON-SIGN CERTIFICATE BUILT?

- Map every model into the same response experiment and metric.
- Compute the smallest covering radius and search the response fiber for the first sign reversal.
- Validate the resulting interval with generated-response uncertainty and the declared structural domain.

WHICH LITERATURES HELP PLACE THE QUESTION?

- Christensen and Connault (2023): robustness of counterfactual conclusions across latent distributions.
- Plagborg-Møller and Wolf (2021): equivalent impulse-response representations under a common target.
- Armstrong, Kline, and Sun (2025): decisions under local misspecification.
- This paper separates family coverage from the first policy-sign reversal under one response protocol.

WHAT IS THE SCOPE OF A REPORTED GAP?

- Each gap belongs to a declared response protocol and weighting.
- The structural domain determines which family members receive coverage.
- The policy target determines where sign reversal occurs.

WHAT SHOULD THE READER CARRY FORWARD?

1. Cross-model agreement is a fiber question.
2. Coverage and sign reversal define separate radii.
3. Experiment choice can move the resulting gap across zero.

TECHNICAL APPENDIX

WHICH DEFINITIONS AND ASSUMPTIONS ORGANIZE THE PAPER?

- Assumption: Common response protocol
- Assumption: Uniform structural-moment approximation
- Definition: Empirical fiber and policy identified set
- Definition: Policy-feature identified set

HOW DO THE MAIN RESULTS BUILD ON ONE ANOTHER?

Global fiber-factorization characterization



Coverage-and-reversal frontier



Experiment-stable sign separation

WHICH FORMAL RESULTS SUPPORT THE ARGUMENT?

- Theorem: Global fiber-factorization characterization
- Theorem: Local factorization characterization
- Theorem: Cross-family identification characterization
- Theorem: Uniform projection inference
- Proposition: Three logically distinct failures
- Proposition: Pointwise differential factorization
- Proposition: Local response metric

WHICH SCOPE CONDITIONS GOVERN THE CLAIMS?

- Each gap belongs to a declared response protocol and weighting.
- The structural domain determines which family members receive coverage.
- The policy target determines where sign reversal occurs.