COUNTERFACTUAL SUFFICIENCY: STATE REPRESENTATION AND RESPONSE GEOMETRY

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WHAT IS THE ECONOMIC QUESTION?

Which additional state coordinates preserve a specified policy calculation?

- A supplied state representation may merge states with different policy responses.
- The relevant loss depends on the declared target, perturbations, and feature class.
- The paper studies exact local repair, approximate compression, and global decoder risk within one state-design contract.

WHAT MUST BE FIXED BEFORE CHOOSING COORDINATES?

- A supplied representation q, policy-response family $\mathcal{R}_\tau$, and target weights W_τ.
- A state distribution, metric, tolerance, and admissible d-coordinate class $\mathcal{B}_d$.
- An evaluation design that keeps coordinate selection and held-out risk assessment separate.

WHEN IS ONE EXTRA COORDINATE ENOUGH?

- Suppose the supplied record merges states along two hidden directions.
- If the declared response changes only along one linear combination, one added coordinate can preserve the local target.
- A second target or a wider perturbation family can raise the restricted derivative rank and the required coordinate count.

WHERE DOES REPRESENTATION LOSS ARISE?

States merged by the record

Different target responses

Decoder risk

WHAT IS THE CANONICAL TARGET QUOTIENT?

Theorem (Target quotient and local repair)
The target response and supplied representation factor through a canonical quotient. At a state, every smooth first-order repair needs at least the rank of the restricted target derivative; a repair of that dimension is attainable locally.

The quotient identifies the distinctions that the declared target still needs after conditioning on the supplied record.

WHAT DOES THE LOCAL COMPRESSION FRONTIER MEASURE?

$$L_d^\star(s) = \sum_{j>d} \lambda_j(s)$$

- The eigenvalues come from the target derivative restricted to directions erased by q.
- Keeping the leading d eigenvectors minimizes quadratic first-order loss.
- The tail sum prices average local loss; λ_{d+1} prices worst-case local loss.

WHAT IS THE BEST RANK-d LOCAL COMPRESSION?

Proposition (Spectral compression frontier)
The leading d eigenvectors of the restricted derivative operator minimize quadratic first-order loss. The residual loss is the sum of discarded eigenvalues, and worst-case first-order loss is governed by λ_{d+1}.

The eigenvalue tail supplies both a coordinate rule and a local effective dimension.

WHY CAN LOCAL DERIVATIVE FIT MISS GLOBAL DECODER RISK?

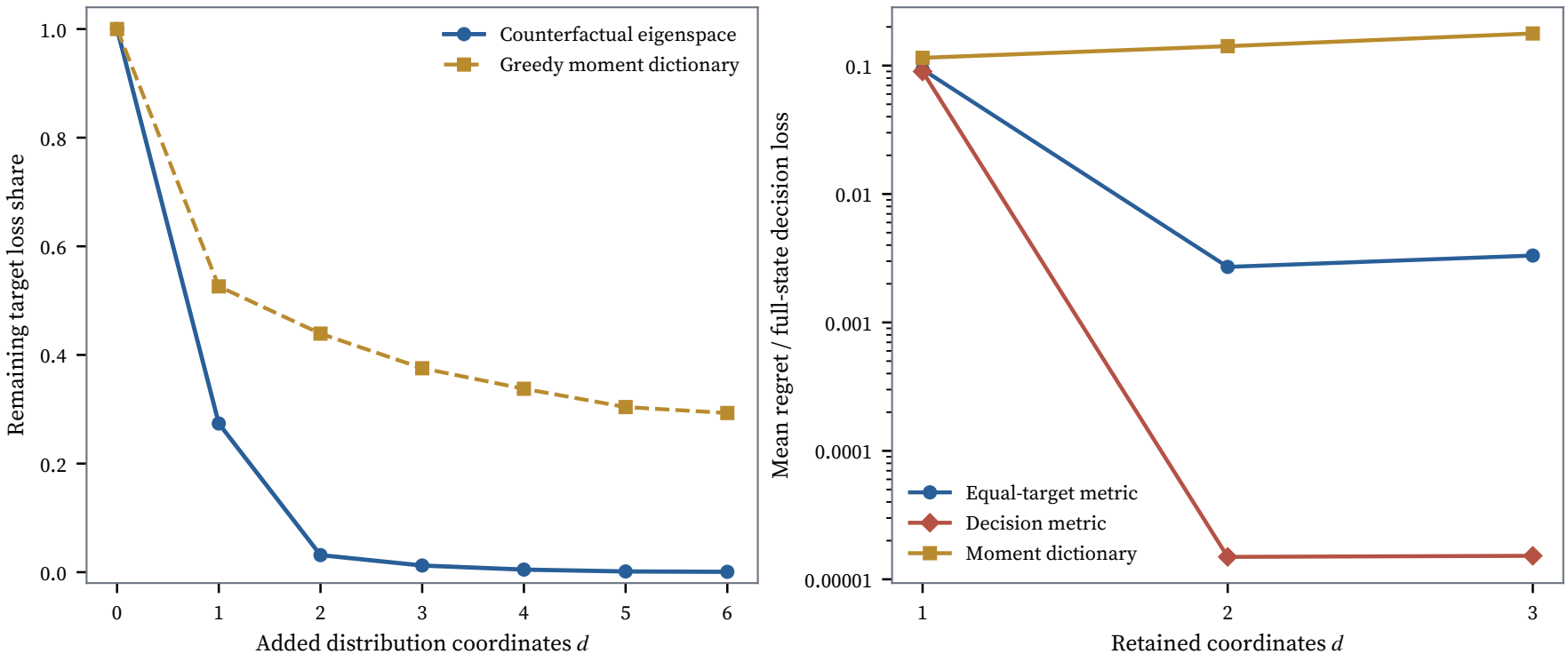
Proposition (Local concentration versus global risk)
For any retained derivative share and any desired risk ratio, a smooth target exists whose leading local derivative subspace retains that share while having decoder risk arbitrarily larger than another coordinate subspace.

Fiber variation enters once the task moves from shrinking perturbations to global prediction.

WHAT DOES THE QUOTIENT LET THE RESEARCHER DESIGN?

- Exact sufficiency becomes a statement about constancy on representation fibers.
- The restricted derivative gives the minimum number of local target coordinates that must survive compression.
- Its spectrum ranks the remaining coordinates when exact recovery exceeds the available capacity.

WHICH DISTRIBUTIONAL COORDINATES MOVE THE POLICY RESPONSE?



Main-text coordinates in the general-equilibrium transition economy.

HOW IS GLOBAL COORDINATE QUALITY ASSESSED?

- A conditional-generator certificate bounds target variation within coordinate fibers.
- Its slack is separated from score estimation, sampling, and optimization error.
- A finite-graph analogue evaluates fitted-decoder residuals on held-out clusters.

HOW IS THE LOCAL LOWER BOUND ATTAINED?

- Choose coordinates whose differentials span the target-relevant quotient at the state.
- For quadratic compression, project onto the leading eigenvectors of the restricted derivative operator.
- The construction is local; the generator and graph criteria evaluate what remains inside global feature fibers.

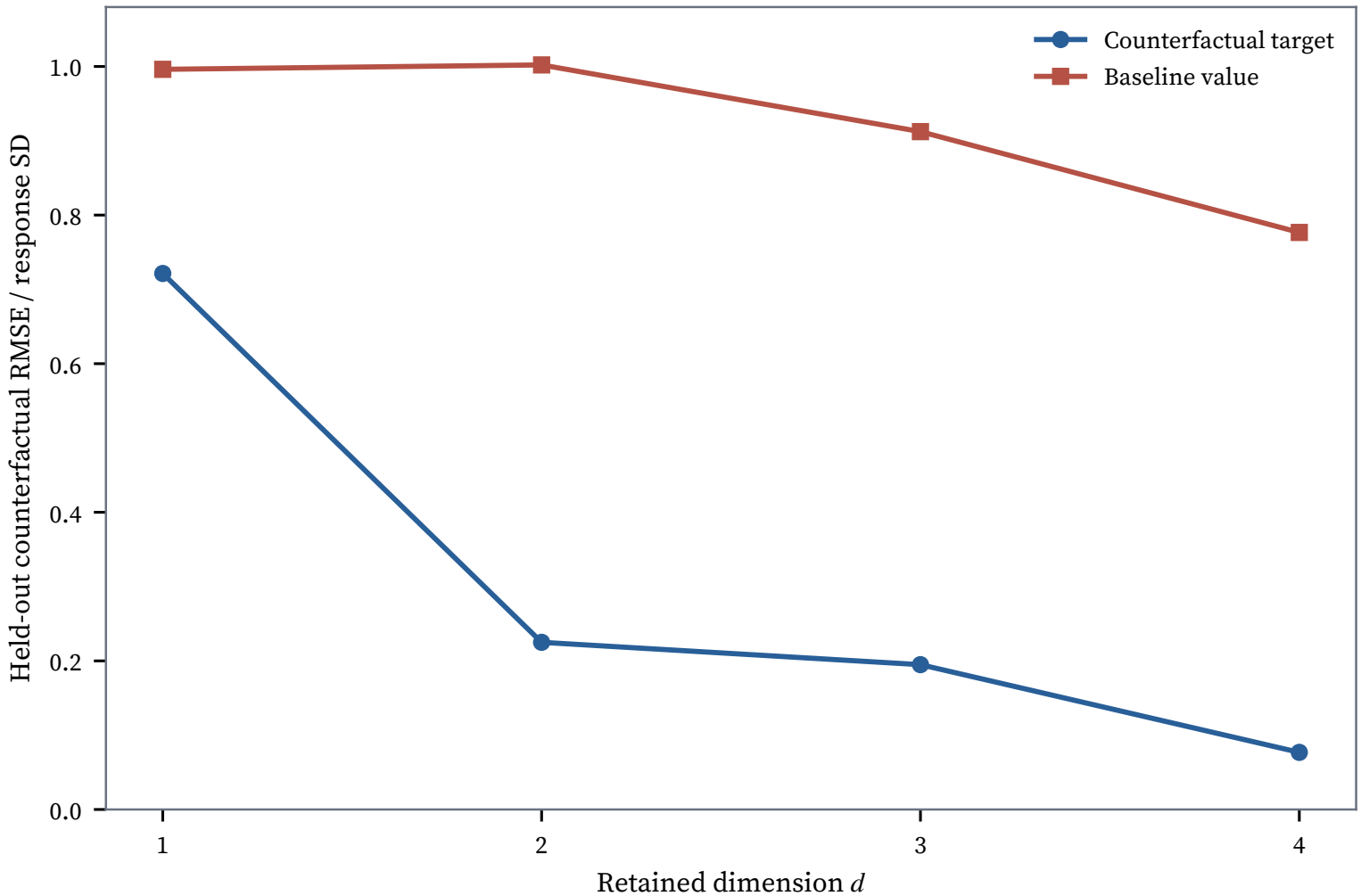
How does coordinate selection avoid reusing the answer?

- An auxiliary sample can construct candidate coordinates, graphs, and decoders.
- Independent evaluation clusters score the selected candidate on held-out target residuals.
- The oracle inequality separates certificate slack, statistical error, and optimization tolerance.

WHAT DO THE EXERCISES COMPARE?

- An occupational-choice model tests whether retained coordinates preserve policy responses.
- A heterogeneous-agent transition economy compares equal-budget coordinate systems.
- The comparisons reuse the same target surface and evaluation protocol across candidate representations.

CAN A HIGH DERIVATIVE SHARE STILL LEAVE LARGE DECODER ERROR?



Appendix adversarial-coordinate comparison across retained dimensions.

WHAT DOES THE CERTIFICATE DIAGNOSTIC ADD?

- Derivative spectra describe shrinking-perturbation loss.
- Generator and graph diagnostics measure variation left inside feature fibers.
- Held-out target risk evaluates the selected coordinate map under the declared state distribution.

WHICH LITERATURES HELP PLACE THE QUESTION?

- Bai and Ng (2002): spectral dimension selection for approximate factor representations.
- Auclert et al. (2021): response Jacobians as a compressed representation of heterogeneous-agent dynamics.
- Hotz and Miller (1993): dynamic state reduction through conditional choice probabilities.
- This paper chooses coordinates relative to a supplied representation, target loss, and admissible feature class.

WHICH CLAIMS REMAIN DISTINCT?

- Representation loss concerns the supplied target and contract.
- Fitted prediction error also contains decoder estimation error.
- Structural identification of the response map requires its own assumptions and data.

WHAT SHOULD THE READER CARRY FORWARD?

1. The target quotient states what the policy calculation needs.
2. Derivative eigenvalues price local compression.
3. Fiber diagnostics govern the move from local geometry to global decoding.

TECHNICAL APPENDIX

WHICH DEFINITIONS AND ASSUMPTIONS ORGANIZE THE PAPER?

- Definition: State-design contract
- Definition: Counterfactual equivalence
- Definition: Loss-relevant counterfactual equivalence

HOW DO THE MAIN RESULTS BUILD ON ONE ANOTHER?

Target quotient and local repair

Spectral compression frontier

Local concentration versus global risk

WHICH FORMAL RESULTS SUPPORT THE ARGUMENT?

- Theorem: Canonical target quotient and minimum local repair
- Theorem: Spectral compression frontier
- Theorem: Local spectral-risk expansion
- Theorem: Exact fiber-certificate gap and sharp bound
- Theorem: Coordinate-selection oracle inequality
- Proposition: Finite-state coordinate count
- Proposition: Kernel of the target operator

WHICH SCOPE CONDITIONS GOVERN THE CLAIMS?

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- Structural identification of the response map requires its own assumptions and data.