Perfectly Still

Stability and Policy Response Capacity

Online Appendix

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Abstract

This appendix collects proofs, additional design and inference details, and supporting results for the randomized applications. It follows the notation, targets, and evidence classifications used in the main text.

JEL Codes: C23, C61, D12, I15, I18

Keywords: policy response capacity, randomized policy history, response matrix, finite policy actions, factorial experiments

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Contents

A. Proofs

A.1. Proof of Proposition 2

PROOF. For each unit, the four values of $(I_{it}, I_{i,t+1})$ give

$$I_{i,t+1}B_{i,t+1} - I_{it}B_{it} = (1 - I_{it})I_{i,t+1}B_{i,t+1} - I_{it}(1 - I_{i,t+1})B_{it} \tag{1}$$
$$+ I_{it}I_{i,t+1}(B_{i,t+1} - B_{it}). \tag{2}$$

Multiplying by ω_i and summing over units yields equation (6). □

A.2. Proof of Proposition 1

PROOF. For a finite action set, evaluating the objective at every $k(a)$ gives equation (4). Population randomization with shares $p_a \geq 0$ and $\sum_a p_a = 1$ generates $\sum_a p_a k(a)$, whose range is $\operatorname{conv}(\mathcal{K})$. Both $\mathcal{K}$ and $\operatorname{conv}(\mathcal{K})$ are contained in $\operatorname{span}(\mathcal{K})$, so minimizing the zero-cost quadratic loss over the span gives a weakly lower value. Equality holds exactly when the set of closest points in the span intersects the relevant attainable set. □

A.3. Proof of Proposition 3

PROOF. Both systems follow $\delta s_{t+1} = A\delta s_t$ under the baseline policy, so their passive trajectories coincide and converge because $\rho(A) < 1$. For $B_0 = 0$, every block in equation (23) is zero. For $B_1 = I_n$, the final block is $CB_1 = C$, which has row rank q. The full horizon matrix therefore has row rank q. □

A.4. Proof of Proposition 4

PROOF. Local Lipschitz continuity and $M_H(s^\star) = 0$ give $\|M_H(s_t)\|_F \leq L\|s_t - s^\star\|$. Exponential attraction gives equation (24). Solving $LK\lambda^t\|s_0 - s^\star\| < \underline{\kappa}$ for t gives the capacity-floor date. □

A.5. Proof of Proposition 1

PROOF. For any bundle S, the condition $W \subseteq V + \operatorname{Im} D_S$ is equivalent to $\pi_V(W) \subseteq \operatorname{span}\{\pi_V(d_\ell) : \ell \in S\}$. Taking S as the full dictionary proves the existence statement. If S is feasible, then

$$d(W, V) = \dim \pi_V(W) \leq \dim \operatorname{span}\{\pi_V(d_\ell) : \ell \in S\} \leq |S|. \tag{3}$$

The lower bound is attained exactly when a set of $d(W, V)$ projected dictionary vectors contains $\pi_V(W)$ in its span. For unrestricted additions, choose vectors $w_1, \ldots, w_d \in W$

whose quotient classes form a basis of $\pi_V(W)$ and activate them at unit cost. Their span with V contains W, so both minima equal d. $\square$

A.6. Proof of Proposition 2

PROOF. The optimizer in equation (19) is $P_V^Q Z$, so the realized residual is $R_V^Q Z$. A Q-orthogonal projector satisfies $(R_V^Q)'Q = QR_V^Q$ and $(R_V^Q)^2 = R_V^Q$. Taking expectations gives

$$L(V) = \frac{1}{2}\mathbb{E}[Z'(R_V^Q)'QR_V^Q Z] = \frac{1}{2}\operatorname{tr}(QR_V^Q\Omega). \tag{4}$$

When $\widetilde{d} \neq 0$, the new component added to the fitted target change is the Q-orthogonal projection of $R_V^Q Z$ onto $\operatorname{span}\{\widetilde{d}\}$. Its squared Q-norm is

$$\frac{(\widetilde{d}'QR_V^Q Z)^2}{\widetilde{d}'Q\widetilde{d}}. \tag{5}$$

The projection theorem makes one half of its expectation equal to the reduction in equation (19). Substitution of $\Omega = \mathbb{E}[ZZ']$ gives equation (21). When $\widetilde{d} = 0$, the addition lies in V and leaves the feasible target-change set unchanged. $\square$

A.7. Proof of Proposition 9

PROOF. Write $x_1 = x(1)$ and $x_0 = x(0)$, with $x_1 \le x_0$. Since $\min\{x, \ell\} = \int_0^\ell \mathbf{1}\{z \le x\}\, dz$, equation (28) gives

$$K(\ell) = -\int_0^\ell \mathbf{1}\{x_1 < z \le x_0\}\, dz. \tag{6}$$

Subtracting the integrals at ℓ_1 and ℓ_0 gives the interval length in equation (29). Applying the identity unit by unit and taking expectations gives equation (30). For incentive-dependent caps, subtract unrestricted use and write

$$\begin{aligned} &\min\{X(1), L_1\} - X(1) - [\min\{X(0), L_0\} - X(0)] \\ &\quad = (X(0) - L_0)_+ - (X(1) - L_1)_+. \end{aligned} \tag{7}$$

Adding and subtracting $(X(1)-L_0)_+$ gives equation (31) because $(X(0)-L_0)_+ - (X(1)-L_0)_+$ equals the displayed interval length. $\square$

A.8. Proof of Proposition 10

PROOF. Write $c = c(a)$. Conditional on v, a household below the first-period cutoff enters Phase 2 without early adoption and purchases when $v \ge p_2$. A household above the cutoff enters with early adoption and purchases when $g \geq p_2 - v$. Hence

$$K_2(a, p_2) = \int_{-\infty}^{c} \mathbf{1}\{v \geq p_2\} f_v(v)\, dv \tag{8}$$

$$+ \int_{c}^{\infty} \Pr\{g \geq p_2 - v \mid v\} f_v(v)\, dv. \tag{9}$$

Leibniz's rule gives the derivative with respect to c. Multiplication by $dc/da = -1$ yields equation (35). □

A.9. Proof of Proposition 11

PROOF. Differentiate equation (38). Equation (36) gives the derivative of current adoption, and equation (35) gives the derivative of later capacity. Equating marginal benefit and marginal cost gives equation (39). □

A.10. Proof of Theorem 1

PROOF. Iteration of the second transition in equation (40) gives $r_t = (1-\alpha)^t r_0$. The population identity gives $x_t = 1 - e - r_t$, and equation (41) gives $K_t = \alpha r_t$. Since $\alpha \in (0,1)$, all three limits in equation (42) follow. At the limit, another campaign acts on a reachable pool of size zero. Its capacity contrast is therefore zero. If $e > 0$, every column generated by the same campaign has zero projection onto changes in the excluded group, so additions within that span leave the gap e unchanged.

With heterogeneous conversion rates, the surviving reachable mass at rate a is $(1-a)^t r_0(a)$ and its next-campaign contribution is $a(1-a)^t r_0(a)$. Integrating gives equation (43), and

$$K_{t+1} - K_t = -\int_0^1 a^2 (1-a)^t r_0(a)\, da \leq 0.$$

The inequality is strict whenever the integrand is positive on a set with positive initial mass. Dominated convergence gives $K_t \to 0$ and $\int_0^1 (1-a)^t r_0(a)\, da \to 0$, because $a > 0$ on the maintained support. Hence the reached share converges to $1 - e$ while the excluded gap remains e. □

A.11. Proof of Proposition 3

PROOF. Sequential assignment and consistency give

$$\mathbb{E}[Y_{t+1+h}(a,b) \mid H_t = h_t] = \int \mathbb{E}[Y_{t+1+h} \mid H_t = h_t, A_t = a, L_{t+1} = \ell, B_{t+1} = b] dF(\ell \mid h_t, a). \tag{10}$$

The smooth-support and envelope conditions permit differentiation with respect to b under the integral, which gives equation (14). A second application of the maintained interchange conditions with respect to a identifies $\theta_h(a, h_t)$. □

A.12. Proof of Proposition 7

PROOF. Joint asymptotic normality and covariance consistency imply convergence of the studentized contrast vector to its Gaussian law. The definition of $c_{1-\alpha}$ gives simultaneous coverage of every component. On that simultaneous-coverage event, the interval chosen by any selector with range in the finite family also covers its contrast. $\square$

A.13. Proof of Proposition 8

PROOF. The availability restriction gives $K_2(a) = \mathbb{E}[Y_2(a,1)]$. Conditional random assignment gives $\mathbb{E}[Y_2(a,1) \mid G] = \mathbb{E}[Y_2 \mid A = a, G]$. Subtracting the expression for a' and averaging over the included blocks gives equation (25). $\square$

A.14. Proof of Proposition 5

PROOF. Conditional random assignment and consistency give $\mathbb{E}[Y_h(d,b) \mid G] = \mathbb{E}[Y_h \mid D = d, B = b, G]$ for each factorial cell. Substitute the four observed conditional means into equation (19), take the two within-configuration intervention contrasts, and average over the included strata. This yields equation (20). $\square$

A.15. Proof of Proposition 6

PROOF. Proposition 5 identifies ΔK_h for every $h \in \{h^\star\} \cup \mathcal{C}$. Apply the weights in equation (22) to these identified cell-mean contrasts. If $\widehat{\theta}$ stacks the corresponding interaction coefficients and a stacks the weights, the estimator is $a'\widehat{\theta}$ and its covariance is $a' \operatorname{Var}(\widehat{\theta}) a$. $\square$

B. Scope Applications

B.1. Sequential bed-net subsidy exposure

The application uses the design and public replication data of Dupas (2014). The experiment introduced Olyset long-lasting insecticide-treated bed nets to 1,120 households in six rural areas of Busia District, Kenya. In Phase 1, households received vouchers with randomized subsidy levels and had three months to redeem them. Four areas entered Phase 2 one year later. All 599 households in those areas received an unannounced voucher offering another Olyset net for 150 Kenyan shillings. The product was unavailable through local retailers during the experiment, while other bed nets remained available. Purchase from the Phase 2 voucher therefore gives the response to a defined later offer.

Define $H_i = \mathbf{1}\{P_{1i} \le 50\}$, where P_{1i} is the randomized Phase 1 price. Let G_i be the share of sampled households within 500 meters whose assigned price also lies at or below 50 shillings. The response targets are D_{1i}, purchase in Phase 1, and D_{2i}, purchase from the Phase 2 offer. For households with Phase 2 outcomes and valid coordinates, the estimated system is

$$\begin{bmatrix} D_{1i} \\ D_{2i} \end{bmatrix} = \alpha_{g(i)} + M^A \begin{bmatrix} H_i \\ G_i \end{bmatrix} + \Gamma Z_i + \varepsilon_i, \tag{11}$$

where $\alpha_{g(i)}$ contains experimental-area effects and Z_i contains the randomized voucher-recipient and marketing-pitch assignments. The first column of M^A is the own-assignment response vector. The second column is the linear exposure-response coefficient for randomized neighborhood assignment.

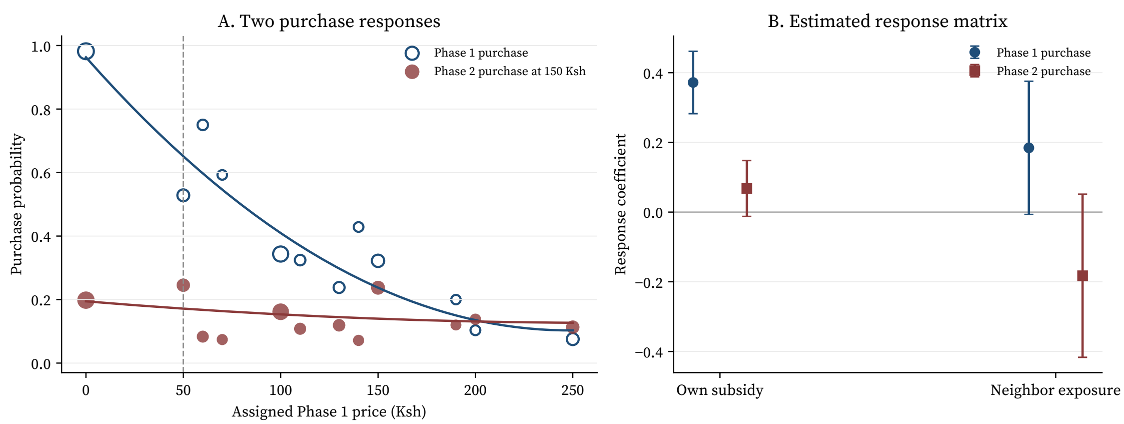

FIGURE 1. Randomized subsidy history and purchase response

Notes. Panel A plots purchase rates by assigned Phase 1 price. Marker area reflects the number of households at each price and lines are observation-weighted quadratic fits. Every Phase 2 household faced a price of 150 Kenyan shillings. Panel B reports the rows of $\widehat{M}^A$ from equation (11); bars are 95 percent intervals from one-kilometer spatial HAC standard errors.

In the 599-household Phase 2 sample, the own high-subsidy coefficient is 0.372 for Phase 1 purchase and 0.0718 for Phase 2 purchase. The within-area randomization values are 0.0001 and 0.0742. Table 1 uses the 584 households with valid coordinates. The covariance estimator stacks the two equation scores and applies a Bartlett spatial kernel to household pairs within one kilometer (Conley 1999).

TABLE 1. Current adoption and later offer response

	Phase 1 purchase	Phase 2 purchase
Own high subsidy	0.372	0.068
	(0.046)	(0.041)
Neighbor high-subsidy share	0.184	-0.183
	(0.098)	(0.119)
Singular values	0.416, 0.193	
Rank-zero Wald test	69.03 ($p < 0.001$)	
Rank-one determinant test	-0.080 (0.047), $p = 0.090$	
Lower-rank-uniform rank-one test	21.86, $p = 0.077$	
Observations	584	

Notes. Each column of the response matrix corresponds to an assigned policy exposure, and each row corresponds to a purchase outcome. High subsidy means a Phase 1 assigned price at or below 50 Kenyan shillings. Neighbor exposure is the share of households within 500 meters assigned a high subsidy. Regressions include experimental-area effects and the randomized voucher-recipient and marketing-pitch assignments. Parentheses report spatial HAC standard errors with a one-kilometer Bartlett kernel. The rank-zero row is a four-degree-of-freedom Wald test. The determinant row uses a delta approximation around a regular rank-one matrix. The lower-rank-uniform row applies the Chen–Fang CFA two-step test with 49,999 Gaussian draws from the spatial-HAC coefficient law. Targets are measured as purchase probabilities and instruments as unit changes in assignment exposure.

The estimated response matrix is

$$\widehat{M}^A = \begin{bmatrix} 0.372 & 0.184 \\ 0.068 & -0.183 \end{bmatrix}. \tag{12}$$

The spatial standard errors for the own column are 0.046 and 0.041. The singular values are 0.416 and 0.193. The rank-zero Wald statistic is 69.03, with four degrees of freedom and $p < 0.001$. The determinant is -0.080 with a delta standard error of 0.047. The Chen–Fang two-step statistic is 21.86 with a Gaussian bootstrap value of 0.077. The rank-zero result identifies one response direction; inference for the second direction crosses the 5 percent threshold.

Among 599 households, 282 purchased from neither offer, 223 purchased only in Phase 1, 38 purchased only in Phase 2, and 56 purchased in both phases. Random assignment identifies the total own-history effect in equation (25). The 7.2-percentage-point later estimate combines experience, ownership, and other pathways through which early assignment changes response to the later offer. The single Phase 2 price identifies this contrast at 150 shillings. A later randomized price schedule would identify the corresponding demand curve.

C. myBPmyLife Micro-Randomized Trial

C.1. Public files and decision sample

Golbus et al. (2026) release the decision-level data, core-cohort identifiers, baseline step counts, codebook, and source analysis files through a public GitHub repository. The replication downloader pins a single repository commit, and its checksum manifest records SHA-256 for the source archive, each downloaded file, and every extracted member. The pinned repository page displays a public download and carries no separate data-license file. The release package therefore carries the downloader, checksums, code, analysis record, and generated exhibits and leaves the source files at their public location.

The core identifier file selects 287 participants and 187,517 recorded randomization points, matching the source article. The active phase contains 186,893 points with a recorded dice roll, spanning relative enrollment days 8 through 180. A successful activity roll occurs at 12.52 percent of these points and a successful diet roll at 12.36 percent. The corresponding design probabilities are 12.5 percent each. The codebook defines `IsPresent_DeviceDataPoints_30minBefore` as recorded app use during the 30 minutes before assignment and `IsPresent_DeviceDataPoints_60minAfter` as recorded app use during the next 60 minutes. The count endpoints record app events, searches, and low-salt choices over the same windows.

C.2. History construction and estimator

Within participant, observations are ordered by relative enrollment day, time of day, preference timestamp row, and dice-roll row. For each window $w \in \{4, 12, 28, 56, 112\}$, the analysis sums successful activity and diet assignments across the preceding w recorded decision points and centers the sum at $0.25w$. A row enters a window specification after all w preceding assignments are observed. The resulting samples contain 185,745, 183,449, 178,857, 170,821, and 154,776 decision points.

Equation (16) is estimated by least squares after absorbing participant, relative-day, and time-of-day effects. The explicit regressors are centered current diet assignment, centered current activity assignment, centered randomized history, both current-assignment-by-history products, the endpoint measured in the preceding 30 minutes, both current-assignment-by-pre-state products, and centered linear and quadratic relative-day terms interacted with both current assignments. Participant-cluster CR1 covariance estimates give pointwise standard errors. Cluster-score vectors are retained for every window, endpoint, and notification type. Multiplying these score vectors by 99,999 Gaussian participant weights and taking the maximum absolute studentized statistic yields the simultaneous critical values and adjusted values.

TABLE 2. Passive app-use transition at control-draw decision points

Quantity	Estimate	Cluster SE
App use after pre-state 0	4.22 pp	0.25 pp
App use after pre-state 1	19.76 pp	1.03 pp
Transition slope	0.155	0.010
Implied two-state fixed point	5.00 pp	0.31 pp

Notes. The sample contains 140,397 active-phase decision points assigned to the 0.75 control condition. The pre-state records app use during the preceding 30 minutes, and the outcome records app use during the next 60 minutes. The first two rows come from a linear transition regression with participant-clustered standard errors. The transition slope is their difference. Iterating the corresponding two-state kernel gives the displayed fixed point.

TABLE 3. Screened randomized-history family

Outcome	Days	Diet history (SE)	Adjusted p	Activity history (SE)	Adjusted p
App use	1	-0.3041 (0.2527)	0.9896	-0.3552 (0.2160)	0.8489
App use	3	-0.1800 (0.1289)	0.9539	-0.1037 (0.1264)	1.0000
App use	7	-0.2745 (0.0947)	0.0878	-0.1278 (0.0830)	0.9040
App use	14	-0.2193 (0.0745)	0.0766	-0.0384 (0.0617)	1.0000
App use	28	-0.0870 (0.0499)	0.7856	0.0209 (0.0486)	1.0000
App events	1	-0.0333 (0.0325)	0.9985	-0.0382 (0.0257)	0.9245
App events	3	-0.0253 (0.0173)	0.9346	-0.0158 (0.0141)	0.9953
App events	7	-0.0330 (0.0129)	0.2069	-0.0145 (0.0091)	0.8814
App events	14	-0.0264 (0.0092)	0.0929	-0.0003 (0.0070)	1.0000
App events	28	-0.0100 (0.0054)	0.7124	0.0027 (0.0053)	1.0000
Searches	1	-0.0029 (0.0058)	1.0000	-0.0051 (0.0049)	0.9985
Searches	3	-0.0047 (0.0034)	0.9522	-0.0029 (0.0025)	0.9942
Searches	7	-0.0050 (0.0024)	0.5295	-0.0016 (0.0016)	0.9984
Searches	14	-0.0034 (0.0017)	0.6226	0.0001 (0.0012)	1.0000
Searches	28	-0.0010 (0.0011)	0.9997	0.0004 (0.0009)	1.0000
Low-salt choices	1	-0.0062 (0.0065)	0.9995	-0.0045 (0.0044)	0.9983
Low-salt choices	3	-0.0034 (0.0034)	0.9988	-0.0019 (0.0028)	1.0000
Low-salt choices	7	-0.0061 (0.0026)	0.3028	-0.0017 (0.0017)	0.9994
Low-salt choices	14	-0.0047 (0.0018)	0.1649	0.0004 (0.0013)	1.0000
Low-salt choices	28	-0.0021 (0.0011)	0.5746	0.0008 (0.0009)	0.9998

Notes. Entries are current-notification-by-randomized-history coefficients with participant-clustered standard errors. App-use coefficients are percentage points per additional past assignment; the three count outcomes use counts per additional past assignment. Adjusted values use the maximum absolute studentized statistic over five windows, four outcomes, and two notification types. Every specification includes the controls and fixed effects listed in Table 1.

The five-coefficient app-use family has a 95 percent simultaneous critical value of 2.493. The 40-coefficient screened family has a critical value of 3.090. For the 14-day history, the app-use diet interaction equals −0.00219 with standard error 0.00074, displayed-family $p = 0.0136$, and screened-family $p = 0.0766$. Its 10th and 90th history percentiles are 10 and 18 assignments. For the 7-day history, the interaction equals −0.00274 with standard error 0.00095 and corresponding adjusted values of 0.0159 and 0.0878. Its history percentiles are 4 and 10 assignments.

The future-history placebo constructs a centered sum from the next 28 randomization points and applies the same current-assignment, pre-state, and relative-time terms. The diet-by-future-history coefficient equals 0.00119 with standard error 0.00094 and $p = 0.206$; the activity coefficient equals 0.00131 with standard error 0.00090 and $p = 0.150$. The result record stores all coefficients, covariance-based standard errors, family values, sample counts, absorption convergence records, source hashes, and transition calculations. `scripts/analyze_mybpmylife.py` recreates Figure 1 and Tables 1, 2, and 3.

D. Digital-Use Experiment and Estimation

D.1. Public files and treatment assignment

The application uses version 2.0 of the Harvard Dataverse deposit for Allcott et al. (2022). The source experiment is registered as AEARCTR-0005796, DOI 10.1257/rct.5796-2.0. The updated Stata sample contains 1,933 randomized participants and 879 variables. The codebook supplies survey and phone-dashboard variable definitions. Source terms permit nonprofit research with citation and prohibit redistribution of the data files. The reproduction code downloads each file from its Dataverse identifier, verifies the posted MD5 digest, and keeps the raw files outside the release archive. The authors also release an MIT-licensed code archive from their Stanford research directory. The machine-readable source manifest records its SHA-256 digest.

Bonus assignment is `S3_Bonus`. Limit configuration is `S2_LimitType`: zero denotes limit control, values one through four correspond to snooze delays of 0, 2, 5, and 20 minutes, and value five denotes a limit without snooze. The binary limit indicator equals one for values one through five. The four bonus-by-limit cells contain 589, 194, 865, and 285 participants. Average daily FITSBY use in period h is `PD_P`h`_UsageFITSBY`. Period 1 supplies baseline use, period 3 is the bonus-payment period, and limit assignment applies from period 2. The released assignment code forms eight strata from median splits of baseline FITSBY use, the addiction index, and the restriction index. It assigns the delayed bonus within strata and then assigns blocker status within stratum-by-bonus cells. Delayed-snooze durations are assigned within the same cells.

D.2. Factorial regression

For periods $h = 2, \ldots, 5$, the analysis estimates

$$Y_{ih} = \alpha_h + \beta_h B_i + \delta_h D_i + \theta_h B_i D_i + \gamma_h Y_{i1} + \lambda_{g(i),h} + \varepsilon_{ih}, \tag{13}$$

where B_i is bonus assignment, D_i is limit assignment, and $\lambda_{g(i),h}$ contains the reported randomization-stratum indicators. Equation (20) gives $K_h(0) = \beta_h$, $K_h(1) = \beta_h + \theta_h$, and $\Delta K_h = \theta_h$. HC2 standard errors provide horizon-specific inference. The joint periods 2–5 test uses the cross-horizon covariance of the four interaction influence functions. The payment-overlap contrast applies weights $(-1/3, 1, -1/3, -1/3)$ to $(\widehat{\theta}_2, \widehat{\theta}_3, \widehat{\theta}_4, \widehat{\theta}_5)$. The simultaneous period family applies the four rows of $(4/3)I_4 - (1/3)\mathbf{11}'$ and uses 200,000 Gaussian draws from the HC2 contrast correlation matrix. The period-1 balance regression removes Y_{i1} from the covariate set.

The weekly-window analysis uses the weekly FITSBY variables for weeks $w = 4, \ldots, 15$ and week 3 as the baseline outcome.[1] It estimates equation (13) on the common 1,903-person sample. Each of ten contrasts assigns weight 1/3 to one consecutive three-week window and weight −1/9 to the other nine weeks. The payment-overlap contrast selects weeks 7–9. Cross-week HC2 influence functions supply the covariance, and 200,000 Gaussian draws supply the maximum statistic.

The raw response matrix stacks β_h and δ_h for periods 2–5 and divides each row by the sample standard deviation of its outcome. Cross-horizon influence functions form the covariance matrix of the eight response coefficients. The bonus exposure schedule is $s_B = (0, 1, 0, 0)'$ and the limit access schedule is $s_D = (1, 1, 1, 1)'$. Let $S = (s_B, s_D)$ and $P_S = S(S'S)^{-1}S'$. The schedule-adjusted matrix is $M^{\perp} = (I - P_S)M$. Applying the same linear transformation to every influence function supplies the covariance of $M^{\perp}$. The raw singular values are 0.821 and 0.293, while the adjusted singular values are 0.094 and 0.024. The lower-rank-uniform test applies equation (26), $\kappa_n = n^{-1/4}$, and 49,999 Gaussian draws from the transformed HC2 covariance. Its value for adjusted rank at most one is 0.066. For the six configuration-specific period-3 responses, a Gaussian max-t critical value is simulated from the HC2 contrast correlation matrix with 200,000 draws and seed 20260829. A second max-t family covers the five response comparisons with limit control.

[1] The source fields are named `PD_WeeklyUsageFITSBY_`w.

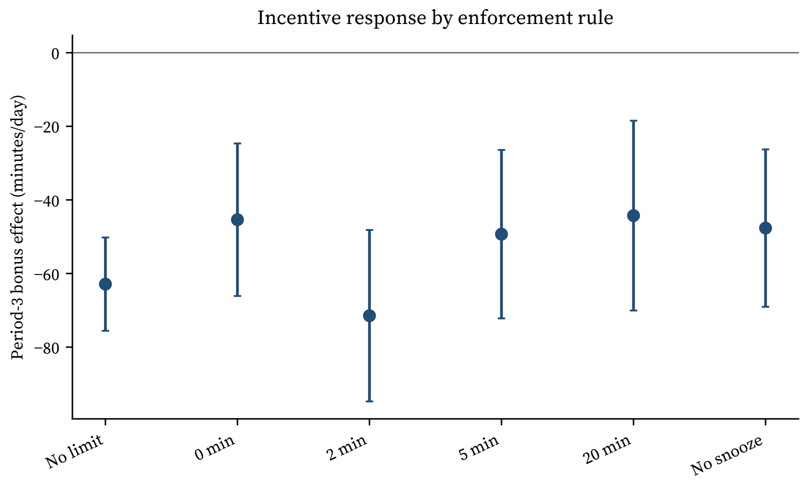

FIGURE 2. Period-3 incentive response by randomized enforcement rule

Notes. Markers report the period-3 bonus effect within the limit-control group and the five randomized limit configurations. Bars give 95 percent Gaussian max-t simultaneous intervals based on the HC2 contrast covariance.

D.3. Author-code reproduction, randomization inference, and planner calculation

The interaction routine estimates equation (13) for $h = 2, \ldots, 5$. The treatment importer defines the limit indicator L, bonus indicator B, stratum factor S, and unit weight. The reproduction calculation re-creates the equation from the deposited data and compares its three treatment coefficients with the implementation used here. The maximum absolute difference across twelve coefficients is 1.7×10^{-10}.

The conditional randomization procedure first permutes bonus assignment within each recorded stratum. Within every new stratum-by-bonus cell, it permutes the detailed limit labels drawn from the corresponding observed cell. This preserves the sequential treatment counts produced by the released assignment algorithm. The procedure keeps the observed outcomes fixed and re-estimates the saturated factorial regression and its HC2 standard error in every draw. The studentized statistic supplies asymptotically valid randomization inference for a weak null of zero average factorial interaction under treatment-effect heterogeneity (Wu and Ding 2021). The period-3 statistic is 1.827 and its 9,999-draw two-sided value is 0.0694 with seed 20260829. A second run uses the common periods 2–5 sample and applies all four one-period-versus-rest contrasts to each assignment draw. The payment contrast has an unadjusted value of 0.0059 and a maximum-studentized family value of 0.0216 with seed 20260830.

For the targeting calculation, Q divides each horizon by its outcome variance in the no-bonus, no-limit assignment cell. A declared common daily use-reduction target r gives $z(r) = (-r, -r, -r, -r)'$. The four feasible target changes are $k(0,0) = 0$, $k(1,0) = \widehat{\beta}$,

$k(0,1) = \widehat{\delta}$, and $k(1,1) = \widehat{\beta} + \widehat{\delta} + \widehat{\theta}$. Equation (4) evaluates these four points. Over targets from zero through 80 minutes, the estimated zero-cost boundaries are 11.61 and 41.53 minutes. At targets of 15, 30, and 50 minutes, the selected actions are limit, limit, and the combined assignment. Gaussian draws use the joint HC2 covariance of all twelve factorial coefficients and select these actions with probabilities 0.998, 1.000, and 0.999. Survey field `S2_PredictUseBonusEarn` supplies a mean expected agency bonus outlay of $76.42. Limit assignment carries zero experimental cash outlay. Fields `S2_MPL` and `S3_MPLLimit` supply the participant valuation exercise. With objective $\lambda L(a) + C(a)$, a 30-minute target selects the limit over no assignment when $c_L < 0.184\lambda$ and over the bonus when $c_L < 76.42 + 0.109\lambda$. At a 50-minute target, the combined assignment beats the limit when $\lambda > 929.9$ under the observed bonus outlay and zero recorded limit outlay.

D.4. Reproduction files

The download script retrieves the source files and writes a checksum manifest. The digital analysis script verifies the author-code archive, writes the coefficient comparison and assignment results, performs the overlap, rank, and targeting calculations, and creates Figures 2 and 2 and Tables 3 and 4. The source article's online appendix reports bonus-by-limit interactions by period. The calculation uses this precedent and maps the coefficients into equations (19) and (22).

E. Bed-Net Experiment and Estimation

E.1. Public replication files

The replication archive for Dupas (2014) was released as `9508_data_and_programs.zip`. Fieldwork ran in 2007–08, before the AEA RCT Registry opened. The link printed by the journal now resolves to a missing page. The analysis uses the archived copy captured from the same Econometric Society URL on January 21, 2022. The source manifest and replication read-me record its SHA-256 digest. The archive contains `main_dataset.dta`, `indiv_level_health.dta`, the original Stata do-file, and the author's read-me.

The household file contains 1,120 rows. The variable `price` is the randomized Phase 1 price. The variables `purchasednet` and `purchasednet2` record redemption from the Phase 1 and Phase 2 shopkeeper logs. Phase 2 purchase is observed for 599 households in four experimental areas. Coordinates are available for 584 of those households. The variables `n_cheap500` and `n_total500` count nearby high-subsidy and sampled households within 500 meters. The analysis defines neighborhood exposure as their ratio and assigns zero when the neighborhood count is zero, matching the construction in the original code.

E.2. Estimation sample and response matrix

Let $Y_i = (D_{1i}, D_{2i})'$ and $T_i = (H_i, G_i)'$. Equation (11) is estimated by least squares on the common 584-household sample. The control vector contains an intercept, three experimental-area indicators, assignment of the voucher to a male respondent or to both respondents, and the burden and cost marketing pitches. The coefficient array is arranged with outcomes in rows and policy exposures in columns.

For coordinates (ϕ_i, λ_i) in radians, pairwise distance is the haversine distance. With cutoff $c = 1$ kilometer, the kernel weight is $w_{ij} = \max\{1 - d_{ij}/c, 0\}$. If X is the equation design matrix, $Q = (X'X)^{-1}$, and e_{ir} is the residual for outcome r, the cross-equation covariance block is

$$\widehat{V}_{rs} = \frac{N}{N-K} Q \left(\sum_i \sum_j w_{ij} x_i x_j' e_{ir} e_{js} \right) Q. \tag{14}$$

Stacking these blocks gives the covariance of the two-equation coefficient vector. This is the covariance used for Table 1 and the rank calculations.

The rank-zero statistic is $\mathrm{vec}(\widehat{M})' \widehat{V}_M^{-1} \mathrm{vec}(\widehat{M})$. For the determinant calculation, the coefficient order is $(m_{11}, m_{12}, m_{21}, m_{22})$ and the gradient is $(m_{22}, -m_{21}, -m_{12}, m_{11})'$. Substitution into equation (27) yields the reported standard error under the regular rank-one approximation. The lower-rank-uniform calculation uses the same coefficient covariance, $\kappa_n = 584^{-1/4} = 0.203$, and 49,999 Gaussian draws. The estimated lower rank is one, the statistic is 21.86, and the bootstrap p-value is 0.0766.

E.3. Randomization inference and transitions

The own-assignment randomization test uses all 599 households with Phase 2 outcomes. The high-subsidy indicator is permuted within each of the four experimental areas, preserving each area's treated count. Every draw residualizes the permuted assignment and each outcome on area effects and the other randomized assignments. Two-sided p-values compare the absolute observed coefficient with 9,999 permuted coefficients and add one to both numerator and denominator. The random seed is 9508.

The observed purchase histories contain 282 households with $(D_1, D_2) = (0, 0)$, 38 with $(0, 1)$, 223 with $(1, 0)$, and 56 with $(1, 1)$. These counts are reproduced directly from `main_dataset.dta`. The analysis script writes the estimation panel, coefficient covariance, transition counts, figure, table, and JSON result record.

E.4. Reproduction files

`scripts/dupas_capacity_analysis.py` reads the public Stata file. It writes a panel CSV and a JSON result record under `data/derived`, along with Figure 1 and Table 1. The original archive and extracted source files remain under `data/raw/dupas2014`.

F. Prospectively Registered Configuration Comparisons

Delgado et al. (2024) registered NCT03833219 and randomized 2,020 drivers among control, feedback, standard incentive, feedback combined with a standard incentive, feedback combined with a reframed incentive, and feedback combined with a doubled incentive. During the 7-week intervention, feedback combined with the standard incentive reduced handheld phone use by 38 seconds per hour relative to control, with a 95 percent interval from 8 to 69 seconds and Holm-adjusted $p = 0.045$. Feedback alone and the standard incentive alone have adjusted values of 0.64 and 0.82. Direct comparisons of the combined arm with these component arms have adjusted values of 0.85 and 0.74. The experiment establishes the combined-arm reduction relative to control and leaves the interaction among the component arms unresolved. During the 6-week follow-up, each active-arm comparison with control has an interval containing zero.

Sachdeva et al. (2026) preregistered AsPredicted 142497 and randomized 149 smartphone users between self-selected and assigned daily-use goals while holding the incentive rule and average goal fixed. Self-selected goals reduced phone use by 26 additional minutes per day and raised goal achievement by 11 percentage points. The assignment changes the configuration through which the common incentive operates. Together, the two trials show how prospective designs can declare a configuration comparison, its outcome horizon, and its analysis family before assignment. The payment-overlap contrast in the present analysis remains exploratory.

G. Additional instruments, value, and baseline stability

G.1. Feasible response addition

Let $V = \operatorname{Im} M$ be the current target span and let $W \subseteq \mathbb{R}^q$ contain target directions required by the policy objective. A response addition is a vector $d \in \mathbb{R}^q$ supplied by an eligibility rule, account system, delivery mode, duration rule, or other available instrument. Let $\mathcal{D} = \{d_1, \dots, d_L\}$ be the available dictionary and let $c_\ell > 0$ be the cost of activating d_ℓ. For

$S \subseteq \{1, \ldots, L\}$, write $D_S = [d_\ell]_{\ell \in S}$ and define

$$k_{\mathcal{D}}(W, V) = \min_{S:\, W \subseteq V + \operatorname{Im} D_S} |S|, \tag{15}$$

$$C_{\mathcal{D}}(W, V) = \min_{S:\, W \subseteq V + \operatorname{Im} D_S} \sum_{\ell \in S} c_\ell, \tag{16}$$

with value $+\infty$ when the constraint has no feasible bundle.

PROPOSITION 1 (Feasible response addition). *Let $\pi_V : \mathbb{R}^q \to \mathbb{R}^q/V$ be the quotient map and let $d(W, V) = \dim W - \dim(W \cap V)$. A feasible bundle exists exactly when*

$$\pi_V(W) \subseteq \operatorname{span}\{\pi_V(d_\ell) : \ell = 1, \ldots, L\}. \tag{17}$$

Every feasible bundle satisfies $|S| \geq d(W, V)$. Equality holds exactly when $d(W, V)$ dictionary vectors have projected span containing $\pi_V(W)$. With unrestricted response vectors and unit activation costs,

$$k_{\mathcal{D}}(W, V) = C_{\mathcal{D}}(W, V) = d(W, V). \tag{18}$$

The dictionary determines the gap between quotient dimension and implementable repair. If available additions project onto a space that excludes one direction in $\pi_V(W)$, repeated activation leaves that target direction outside the response span. If the dictionary spans the missing space through vectors carrying additional components, the minimum feasible bundle can contain more than $d(W, V)$ elements.

G.2. Option value of a response direction

Let $Z \in \mathbb{R}^q$ be a future target shortfall with finite second moment $\Omega = \mathbb{E}[ZZ']$, and let Q be a positive-definite target-weight matrix. For continuously scalable instruments, a planner with response span V chooses an implementable target change $v \in V$ after observing Z and incurs

$$\ell(V, Z) = \min_{v \in V} \frac{1}{2}(Z - v)' Q (Z - v). \tag{19}$$

Let P_V^Q be the Q-orthogonal projector onto V, and write $R_V^Q = I - P_V^Q$. The ex ante loss is $L(V) = \mathbb{E}[\ell(V, Z)]$.

PROPOSITION 2 (Response option value). *The expected targeting loss satisfies*

$$L(V) = \frac{1}{2} \operatorname{tr}(Q R_V^Q \Omega). \tag{20}$$

For a response addition d, define $\widetilde{d} = R_V^Q d$. If $\widetilde{d} \neq 0$, its option value is

$$L(V) - L(V + \operatorname{span}\{d\}) = \frac{1}{2}\frac{\widetilde{d}'QR_V^Q\Omega(R_V^Q)'Q\widetilde{d}}{\widetilde{d}'Q\widetilde{d}}. \tag{21}$$

An addition with $\widetilde{d} = 0$ has value zero in equation (19).

Equation (21) assigns value to the component of a delivery option that lies outside the current response span. The target weights and second moments determine which projected direction absorbs more expected loss. Activation costs from equation (15) convert this value into a net-benefit comparison over feasible bundles. Rank and singular values describe the response map under reported units; Q, Ω, and the activation costs complete the planner's ordering.

G.3. Baseline dynamics and intervention dynamics

For a local linear system,

$$\delta s_{t+1} = A\delta s_t + B\delta u_t, \qquad \delta y_t = C\delta s_t, \tag{22}$$

the horizon response is

$$M_H = C\begin{bmatrix} A^{H-1}B & A^{H-2}B & \cdots & B \end{bmatrix}. \tag{23}$$

Powers of A govern baseline convergence. The image of equation (23) governs policy response capacity. The adoption model below derives a transition for that image from adoption and remaining reach.

PROPOSITION 3 (Baseline convergence leaves response rank unrestricted). *Let $A \in \mathbb{R}^{n\times n}$ satisfy $\rho(A) < 1$, and let $C \in \mathbb{R}^{q\times n}$ have row rank q. Two systems can share A, C, and every baseline trajectory while their horizon response ranks equal zero and q. In particular, $B_0 = 0$ gives* $\operatorname{rank} M_H = 0$, *while $B_1 = I_n$ gives* $\operatorname{rank} M_H = q$ *for every $H \geq 1$.*

The two systems generate the same observations under $u_t = 0$. Their intervention laws differ through B. Baseline convergence therefore supplies a law for passive trajectories, while policy experiments and maintained restrictions on B supply information about response rank.

For a nonlinear baseline law $s_{t+1} = F_0(s_t)$, define continuous response strength by $\kappa_H(s) = \|M_H(s)\|_F$.

PROPOSITION 4 (Convergence to perfect stillness). *Suppose $s^\star$ has a neighborhood on which $\|F_0^t(s) - s^\star\| \leq K\lambda^t\|s - s^\star\|$ for $K < \infty$ and $\lambda \in (0,1)$. If M_H is locally Lipschitz with constant L*

and $M_H(s^\star) = 0$, then every baseline trajectory in that neighborhood satisfies

$$\kappa_H(s_t) \le LK\lambda^t \|s_0 - s^\star\|. \tag{24}$$

For a capacity floor $\underline{\kappa} > 0$, the trajectory enters $\{s : \kappa_H(s) < \underline{\kappa}\}$ by every integer date greater than $\log(LK\|s_0 - s^\star\|/\underline{\kappa})/(-\log\lambda)$.

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