

# SUBJECT TO AVAILABILITY: OPTION PRODUCTION UNDER COMPLEMENTARY INPUTS

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# WHAT IS THE ECONOMIC QUESTION?

How do policy packages create actionable options from complementary inputs?

- An option becomes actionable only when a sufficient path of inputs is supplied.
- Observed choices combine option production, consideration, and preference.
- The paper recovers and bounds the minimal input paths behind package surfaces.

## WHAT DOES A TWO-INPUT REPAIR SET MEAN?

- A job may require care to open early and close late enough for the full shift.
- Supplying either timing block alone leaves the job infeasible; supplying both activates it.
- The job contributes mass to the exact repair set  $\{\text{early opening, late closing}\}$ .

## WHAT IS AN OPTION-PRODUCTION SYSTEM?

- A package supplies a subset of inputs such as opening, closing, transport, or cash support.
- A minimal path is an antichain of input sets that can activate an option.
- The package surface records the mass of options made actionable by each supplied package.

## HOW DOES A PACKAGE SURFACE REVEAL REPAIR MASS?

$$M(S) = \sum_{T \subseteq S} \theta(T)$$

- $M(S)$  is the option mass activated by package  $S$ .
- $\theta(T)$  is the mass whose exact repair set is  $T$ .
- Möbius inversion recovers  $\theta(T)$  when the full exact-repair surface is observed.

## WHEN DOES A PACKAGE SURFACE ADMIT A PATH REPRESENTATION?

### **Theorem (Minimal-path representation)**

*A package surface has a minimal-path representation exactly when it is monotone. A representation supported on one exact repair set per option exists exactly when all Möbius coefficients are nonnegative; those singleton-path masses are then unique.*

Monotonicity governs general paths, while nonnegative Möbius coefficients govern unique exact repair sets.

## WHAT CAN INCOMPLETE PACKAGE SUPPORT IDENTIFY?

### **Proposition (Sharp repair bounds)**

*Observed package masses and nonnegative repair masses define a linear feasible set. Minimizing and maximizing any declared linear functional over that set gives sharp bounds; point identification is determined by the row space of the observed-package incidence matrix.*

The same feasible set supports simultaneous projection inference for prespecified functionals.

## HOW ARE SHARP BOUNDS COMPUTED UNDER INCOMPLETE SUPPORT?

- Stack observed package masses as linear restrictions on nonnegative repair masses.
- Minimize and maximize the prespecified repair functional over the feasible polytope.
- Projection of a joint confidence set carries sampling uncertainty into the same linear programs.

## WHAT CAN PACKAGE VARIATION REVEAL ABOUT OPTION PRODUCTION?

- Each unit is indexed by the smallest input set that turns a latent action into an available option.
- The full package surface recovers exact repair masses by Möbius inversion.
- Incomplete package support yields sharp linear-program bounds for prespecified repair and policy functionals.

# WHY DO COMPLEMENTARY INPUTS CHANGE PACKAGE EFFECTS?

**Supplied inputs**



**Minimal repair paths**



**Actionable options and choice**

## HOW ARE OPTION-PRODUCING INPUTS PRICED?

### **Proposition (Minimum enablement duality)**

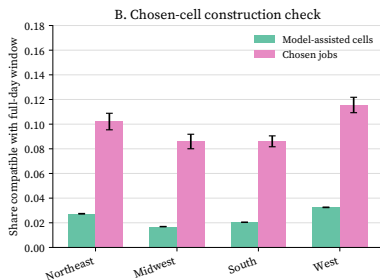
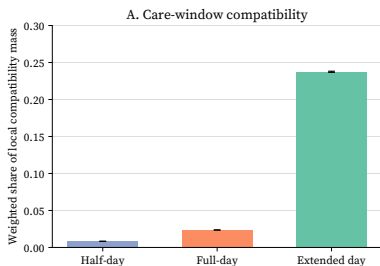
*The minimum-cost enablement program has a dual whose multipliers price the supplied constraints. At an optimum, expenditure equals the dual value and active inputs satisfy the corresponding shadow-price condition.*

The dual assigns the cost of option production across providers and constraints.

## HOW ARE CHILDCARE AND JOB COMPATIBILITY CONNECTED?

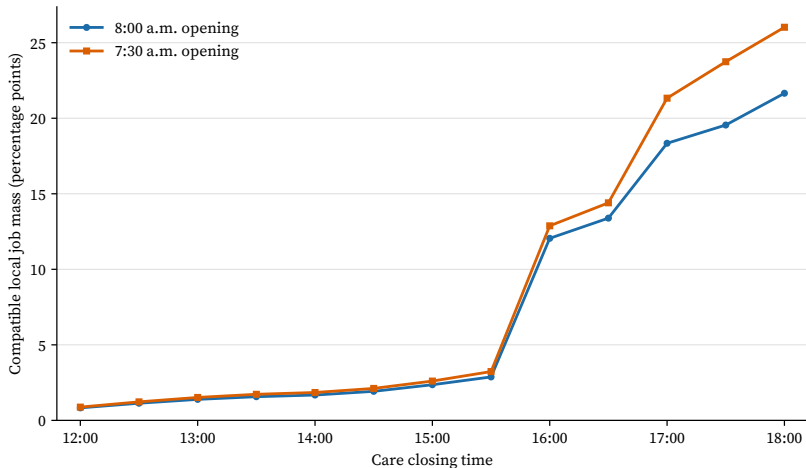
- ACS schedules map care windows into model-assisted job-compatibility mass.
- A childcare-cash factorial supplies package variation in Uganda.
- A cross-fitted transport design links the trial to the 2019–2020 Uganda National Panel Survey.

# HOW MUCH JOB COMPATIBILITY DOES A LONGER CARE WINDOW PRODUCE?



Main-text care-window and job-compatibility benchmark.

# WHERE DOES THE CARE-WINDOW COVERAGE FRONTIER BEND?



Main-text coverage frontier over opening and closing times.

## WHICH MEASUREMENTS DISTINGUISH PRODUCTION FROM USE?

- Direct active-set measurements describe option production.
- Randomized package outcomes describe realized use and behavioral response.
- Population transport and provider accounts address external validity and resource provision.

## HOW DO FOUR EVIDENCE LAYERS ANSWER DIFFERENT QUESTIONS?

- ACS schedules measure model-assisted option production under care windows.
- The randomized factorial measures realized package response in the trial.
- Population transport and provider accounts address the supported target population and resource cost.

## WHAT DO THE CHILDCARE APPLICATIONS FIND?

- Job-compatibility mass rises from 0.83% under 8 a.m.–noon to 23.74% under 7:30 a.m.–5:30 p.m.; 93.1% of repaired mass needs at least two timing blocks.
- The factorial bounds short-term response mass attributable to either supply route between 3.17 and 5.15 percentage points.
- The childcare-factor effect on full-day use is 0.442 in the trial and 0.429 in the supported target at short term; long-term effects are 0.313 and 0.204.

## WHICH LITERATURES HELP PLACE THE QUESTION?

- Mbiti et al. (2019): experimental complementarity across educational inputs.
- Kline and Walters (2016): program access, substitution, take-up, and welfare.
- Gelbach (2002): public childcare access and maternal labor supply.
- This paper represents the minimal input paths that produce an actionable option before choice.

## WHICH OBJECTS SHOULD REMAIN SEPARATE?

- Feasibility describes whether an option is actionable.
- Consideration and choice describe behavior after an option exists.
- Costs, transfers, preferences, and welfare require their own accounting and assumptions.

## WHAT SHOULD THE READER CARRY FORWARD?

1. Complementary inputs produce options through minimal repair paths.
2. Package support determines point identification or sharp bounds.
3. Childcare evidence separates option production, use, transport, and provider cost.

# TECHNICAL APPENDIX

# HOW DO THE MAIN RESULTS BUILD ON ONE ANOTHER?

**Minimal-path representation**



**Sharp repair bounds**



**Minimum enablement duality**

## WHICH FORMAL RESULTS SUPPORT THE ARGUMENT?

- Theorem: Minimal-path representation theorem
- Theorem: Package-surface characterization
- Theorem: Sharp repair bounds
- Proposition: Minimum enablement duality
- Proposition: Repair-set inversion
- Proposition: Choice-equivalent availability systems
- Proposition: Repair-targeting restrictions

## WHICH SCOPE CONDITIONS GOVERN THE CLAIMS?

- Feasibility describes whether an option is actionable.
- Consideration and choice describe behavior after an option exists.
- Costs, transfers, preferences, and welfare require their own accounting and assumptions.