

THE POLAR EXPRESS: DYNAMIC AGGREGATION UNDER POLICY HOLONOMY

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WHAT IS THE ECONOMIC QUESTION?

Can a policy cycle restore an aggregate report while changing the response to a later intervention?

- A closed path returns the reported aggregate to its starting value.
- The transported hidden state may still move along the fiber of that report.
- A common future probe reveals whether the policy response has changed.

HOW CAN A CLOSED AGGREGATE LOOP LEAVE AN OPEN RESPONSE?

Closed report path



Transported hidden state

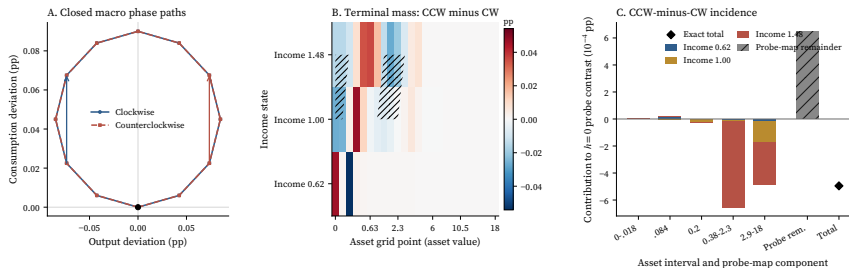


Changed future response

HOW CAN A CLOSED LOOP CHANGE A LATER RESPONSE?

- A clockwise and a counterclockwise policy path can both return output and consumption to the same report.
- The two paths can rearrange the hidden distribution in opposite directions along the report fiber.
- Applying the same terminal probe reveals the resulting difference in future consumption response.

WHERE CAN A CLOSED REPORT PATH LEAVE THE HIDDEN STATE?



Main-text schematic of the report path, transported state, and target defect.

WHAT DEFINES TARGET HOLONOMY?

- A feasible report path and a specified policy technology lift the path through hidden state space.
- The terminal state receives the same future probe as the initial state.
- The change in the probe response is target holonomy.

WHAT DOES REVERSING A SMALL LOOP IDENTIFY?

Theorem (Direction reversal and target curvature)

For oppositely oriented shrinking policy rectangles, the leading difference in terminal target response is governed by the target curvature generated by the policy directions.

Direction reversal removes common lower-order movement and isolates the local path effect.

WHAT DOES DIRECTION REVERSAL ISOLATE?

$$\frac{\psi(\Phi_{\varepsilon}^{+}(s)) - \psi(\Phi_{\varepsilon}^{-}(s))}{2\varepsilon^2} = \Omega_{\psi}(s; v, w) + O(\varepsilon)$$

- Φ_{ε}^{+} and Φ_{ε}^{-} are opposite loop orientations.
- ψ is the common terminal target probe.
- Ω_{ψ} is the local target curvature left after first-order movement cancels.

WHAT DOES A POLICY LOOP REVEAL ABOUT AGGREGATION?

- A closed loop can return the reported aggregate to its starting value while changing the hidden distribution.
- Opposite loop orientations isolate the resulting second-order target response.
- The target curvature supplies a lower bound, and under the stated moment conditions a construction, for the measurements needed to close the aggregate state.

WHAT DESIGN IDENTIFIES THE LOOP CONTRAST?

- Random assignment chooses loop orientation or path cells.
- A common terminal probe is applied after the assigned path.
- Shrinking-loop inference separates curvature from finite-loop remainder under the stated rate conditions.

HOW MANY ADDITIONAL MEASUREMENTS CLOSE THE RESPONSE?

Proposition (Minimum measurement dimension)

On an observational fiber, the rank of the target response map gives the local number of hidden directions that additional measurements must separate; coordinates spanning those directions attain the bound.

The measurement count belongs to the declared future-probe family.

HOW IS THE MINIMUM MEASUREMENT DIMENSION ATTAINED?

- Restrict attention to the observational fiber of the supplied aggregate report.
- Measure a basis for the target-response image left on that fiber.
- When the basis consists of additive moment differentials, additive measurements attain the local bound.

WHEN DOES A DYNAMIC AGGREGATE CLOSE LOCALLY?

Proposition (Dynamic aggregation characterization)

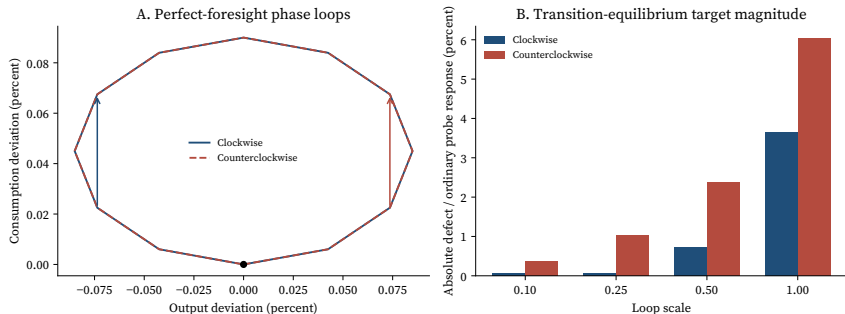
A phase-conditional Gorman representation closes the target locally when the relevant response covectors lie in the span generated by the reported state and additive internal coordinates.

The condition links an aggregation representation to the future responses it preserves.

WHAT DOES THE TRANSITION ECONOMY HOLD FIXED?

- The constructed economy has 72 states and a declared output-consumption report.
- The policy loop closes in the reported coordinates before the common consumption probe.
- Public delivery records form a separate predictive-closure exercise.

HOW LARGE IS THE FUTURE-RESPONSE DEFECT IN THE TRANSITION ECONOMY?



Main-text policy-holonomy calculation in transition equilibrium.

WHAT IS THE SIZE OF THE CONSTRUCTED HOLONOMY?

- A closed output-consumption loop changes the later consumption response by as much as 0.00714 percentage point.
- That change equals 6.04 percent of the ordinary response in the stated comparison.
- The magnitude belongs to the declared loop, terminal probe, and 72-state economy.

HOW WOULD AN EXPERIMENT IDENTIFY HOLONOMY?

- Randomize feasible path cells or loop orientation.
- Apply the same probe after each path and compare terminal responses.
- Shrink loop area or model the finite-loop remainder to interpret the contrast as local curvature.

WHICH LITERATURES HELP PLACE THE QUESTION?

- Gorman (1968): conditions under which heterogeneous objects aggregate exactly.
- Auclert et al. (2021): sequence-space response maps in heterogeneous-agent models.
- Liang (2022): dynamically aggregated information and sufficient summaries.
- This paper studies path dependence that survives after a reported aggregate returns to its starting point.

WHICH EVIDENCE ADDRESSES MECHANISM AND WHICH ADDRESSES PREDICTION?

- The transition economy demonstrates the dynamic aggregation mechanism.
- Randomized loops with common probes identify the experimental holonomy contrast.
- Public delivery records assess predictive closure under their observed support.

WHAT SHOULD THE READER CARRY FORWARD?

1. Returning an aggregate to its initial value can leave future response changed.
2. Loop reversal identifies local target curvature.
3. Additional measurement is priced by the rank of the response left on the fiber.

TECHNICAL APPENDIX

WHICH DEFINITIONS AND ASSUMPTIONS ORGANIZE THE PAPER?

- Assumption: Constant-rank policy technology
- Definition: Loop sufficiency

HOW DO THE MAIN RESULTS BUILD ON ONE ANOTHER?

Direction reversal and target curvature



Minimum measurement dimension



Dynamic aggregation characterization

WHICH FORMAL RESULTS SUPPORT THE ARGUMENT?

- Theorem: Direction reversal and target curvature
- Theorem: Shrinking-loop identification and inference
- Theorem: Cluster-assigned path identification
- Theorem: Local-holonomy power frontier
- Theorem: Curvature bridge and residual sequence
- Theorem: Minimum measurement dimension on an observational fiber
- Theorem: Dynamic aggregation characterization

WHICH SCOPE CONDITIONS GOVERN THE CLAIMS?

- The transition economy demonstrates the dynamic aggregation mechanism.
- Randomized loops with common probes identify the experimental holonomy contrast.
- Public delivery records assess predictive closure under their observed support.